

Title: Constraint satisfaction problems and compactness

Abstract: The Constraint Satisfaction Problem, or CSP for short, over a relational structure \mathfrak{B} is the decision problem where the input is a finite structure \mathfrak{A} with the same signature as \mathfrak{B} and we need to decide whether there exists a homomorphism from \mathfrak{A} to \mathfrak{B} . According to the famous result of Bulatov and Zhuk we know that CSPs over finite structures exhibit a complexity dichotomy: they are all in \mathbf{P} or \mathbf{NP} -complete.

In the past few years some interesting connections have been discovered between CSPs and compactness principles in choiceless set theory. For a structure \mathfrak{B} we write $K_{\mathfrak{B}}$ for the following statement: for every structure \mathfrak{J} with the same signature as \mathfrak{B} , if all finite substructures of \mathfrak{J} homomorphically map to \mathfrak{B} then so does \mathfrak{J} . The general observation is that the harder the CSP over some structure \mathfrak{B} is the stronger the compactness principle $K_{\mathfrak{B}}$ is (over ZF). In my talk I will talk about some recent results supporting this observation including a gentle introduction to the theory of CSPs.