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Some Ramsey theory and topological dynamics for first order theories

Abstract. I will discuss a theory developed in my joint paper with Junguk Lee and Slavko Moconja. One can view it as a variant of Kechris, Pestov, and Todorćević theory in the context of (complete first order) theories. I will discuss several “definable” Ramsey-theoretic properties of first order theories and their dynamical characterizations. The point is that all the Ramsey-theoretic properties that we introduce involve “definable colorings” and the dynamical characterizations are “dynamical properties of the theories”, i.e. they are expressed in terms of the action of the group of automorphisms of a monster (i.e. sufficiently saturated and homogeneous) model of the theory in question on the appropriate space of types. One of the basic results says that a theory has the definable Ramsey property iff it is extremely amenable (as defined by Hrushovski, Pillay and myself). But there are various other results, some of which are essentially new and may be surprising in comparison with the Kechris, Pestov, Todorćević theory. One of the motivations to study those properties was to understand better the so-called Ellis group of a theory (which was used by Pillay, Rzepecki, and myself to explain the nature of the Lascar Galois groups of first order theories and spaces of strong types, and led E. Hrushovski to some original development with striking applications to approximate subgroups). Using our dynamical characterizations, we obtain several criteria for profiniteness and for triviality of this Ellis group, with many examples where they apply. I will try to discuss it during my talk. If time permits, I may very briefly mention an abstract generalization of the above considerations and results, which also applies both to the context of definable groups as well as to the classical context of Kechris Pestov, Todorćević theory, leading to some new notions, results and questions.