

Complex analysis and geometry: celebrating the 70+1 birthday of László Lempert

Program

Monday, 26th June

- 9:00-9:55

Registration

- 9:55-10:00

Welcome

- 10:00-10:50

Duong Phong

Diameters and Green's functions in Kähler geometry

Estimates for the diameter and the Green's function for Kähler metrics are established which require only an entropy bound and no lower bound on the Ricci curvature. The proof builds on recent PDE techniques for L^∞ estimates for the Monge-Ampère equation, with a key improvement allowing degeneracies of the volume form along subvarieties of codimension strictly greater than one. As a consequence, we solve the long-standing problem of uniform diameter bounds and Gromov-Hausdorff convergence of the Kähler-Ricci flow, for both finite-time and long-time solutions. This is joint work with B. Guo, J. Song, and J. Sturm.

- 11:30- 12:20

Mei-Chi Shaw

The Cauchy-Riemann Equations on the Hartogs Triangles

The Hartogs triangle in the complex Euclidean space is an important example in several complex variables. It is a bounded pseudoconvex domain with non-Lipschitz boundary. In this talk, we discuss the extendability of Sobolev spaces on the Hartogs triangle and show that the weak and strong maximal extensions of the Cauchy-Riemann operator agree. These results are related to the Dolbeault cohomology groups with Sobolev coefficients on the complement of the Hartogs triangle (joint work with A. Burchard, J. Flynn and G. Lu).

- 14:00-14:50

Chinh H. Lu

Complex Monge-Ampère equations on compact hermitian manifolds

We discuss a new approach to the uniform Linfty bound for solutions to complex Monge-Ampère equations on compact manifolds. Our method applies in degenerate contexts when the reference form is semi-positive and big, allowing to obtain bounded solutions when the right-hand side is in L^p for some $p > 1$. This is a joint work with Vincent Guedj.

- 15:30-16:20 Erlend Fornaess Wold

Oka properties of complements of unbounded convex sets

We will discuss results related to the following, obtained by the speaker and F. Forstneric: Suppose that $E \subset C^n, n \geq 2$, is a closed convex set that does not contain a real line. Then $C^n \setminus E$ is an Oka domain.

- 16:30-17:10

Lightning talks

Ahmed Yekta Okten: Observations on (non) visibility with respect to Kobayashi geodesics

Roberto Albesiano: A deformation approach to Skoda's division theorem

Johan Jacob Klemensen: A Liouville Theorem and C^α -Estimate for Calabi-Yau cones

Sze Hong Kwong: Higgs bundles and local systems

- 17:10-18:10

Wine and Cheese

Tuesday, 27th June

- 10:00-10:50

Eleonora di Nezza

Regularity of envelopes

Envelopes are key objects in pluripotential theory which also play a central role in analytic proof of results of algebraic geometric nature. In this talk I will introduce envelopes on a compact Kähler manifold and I will study their regularity. This is based on a joint work with S. Trapani.

- 11:30-12:20

Yanir Rubinstein

L^p -polarity, Mahler volumes, and Bourgain's slicing conjecture

The polar K° and the support function h_K of a convex body K are fundamental objects in Functional and Convex Analysis. The Mahler and Bourgain Conjectures have motivated an enormous amount of research in those fields over the past 85 years. Motivated by Bergman kernels, in joint work with B. Berndtsson and V. Mastrantonis we point out that K° and h_K are L^∞ versions of a more general one-parameter family of objects and introduce the associated one-parameter generalization of the Mahler volume and Conjecture, with a subtle and possibly crucial advantage over the original conjecture: uniqueness of extremizers. We settle the upper bound by using Ball's Brunn–Minkowski inequality for harmonic means, the classical Brunn–Minkowski inequality, symmetrization, and a systematic study of the L^p -Mahler volumes. We also explore a new connection between these objects and Bourgain's slicing conjecture motivated by complex geometry. coming back full circle to Bergman kernels.

- 14:00-14:50

Franz Forstneric

Oka-1 manifolds

We introduce a new class of complex manifolds: Oka-1 manifolds. They are characterized by the property that holomorphic maps from any open Riemann surface satisfy the Runge approximation and the Weierstrass interpolation condition. We prove that every complex manifold which is dominable by tubes of complex lines is an Oka-1 manifold. In particular, a manifold dominable by \mathbb{C}^n at most points is an Oka-1 manifold. This provides many examples of Oka-1 manifolds among compact algebraic surfaces, including all Kummer and all elliptic K3 surfaces. We also show that every compact rationally connected manifold is an Oka-1 manifold. The class of Oka-1 manifolds is invariant under Oka maps inducing a surjective homomorphism of fundamental groups; this includes holomorphic fibre bundles with connected Oka fibres. In another direction, we prove that every bordered Riemann surface admits a holomorphic map with dense image in any connected complex manifold.

- 15:30-16:20

Francine Meylan

Polynomial models with few infinitesimal symmetries

One possible approach to study local CR equivalence of real hypersurfaces in \mathbb{C}^{n+1} is the “Taylor expansion” approach: it involves the assignment of different weights to different directions. When the hypersurface is of finite type m (in the sense of Kohn and Bloom-Graham) at a given point, the complex normal coordinate is assigned the weight 1 while the n complex tangential coordinates are assigned the weight $\frac{1}{m}$. They allow to define a homogeneous model (hypersurface) for which the Lie algebra of infinitesimal symmetries admits a natural grading. Since the kernel of the generalized Chern–Moser operator corresponds to the above Lie algebra, this gives a tool for addressing the equivalence problem. We point out that full classification of such Lie algebras seems still out of reach, due in particular to the presence of a component containing nonlinear rigid vector fields, with arbitrarily high degree coefficients. In this talk, we will focus on a class of Levi degenerate hypersurfaces for which the structure of the Lie algebra of infinitesimal CR automorphisms of their models is very simple, leading to second jet determination for their automorphisms.

- 16:30-17:00

Lightning talks

Kuang-Ru Wu: Positively curved Finsler metrics on vector bundles

Prakhar Gupta: A complete metric structure on the space of low energy potentials

Melody Wolff: Asymptotics of Fubini-Study currents for sequences of line bundles

Wednesday, 28th June

- 10:00-10:50

Gábor Székelyhidi

Regularity for singular Kähler-Einstein metrics

Singular Kähler-Einstein metrics arise naturally when studying limits of sequence of smooth Kähler-Einstein manifolds. Through the work of Donaldson-Sun and Li-Wang-Xu we know that the tangent cones of such singular KE metrics are determined by the underlying complex variety, however it is important to have more refined geometric information. For certain classes of isolated singularities, such as ordinary double points, Hein-Sun provided a precise asymptotic description. I will discuss results extending this work to singularities with more general tangent cones, including those with non-isolated singular sets, as well as "unstable" examples where the tangent cone is not locally biholomorphic to the original complex variety. This is joint work with Shih-Kai Chiu.

- 11:30-12:20

Bo Berndtsson

A tour through László's mathematics

Thursday, 29th June

- 10:00-10:50

Mattias Jonsson

Monge–Ampère equations on the boundary of a simplex, with applications to the SYZ conjecture

I will report on joint work with Hultgren, Mazzon, and McCleerey, when we study the real Monge–Ampère equation on the boundary of the unit simplex. Through the work of Yang Li, this has applications to the metric SYZ conjecture.

- 11:30-12:20

Sebastien Boucksom

Measures of finite energy in complex and non-Archimedean pluripotential theory

I will present a joint work with Mattias Jonsson, in which we develop a synthetic approach to global pluripotential theory, covering both the complex analytic and non-Archimedean settings. This allows in particular to study of the dependence of the energy of a measure with respect to the polarization, with applications to cscK metrics and K-stability.

- 14:00-14:50

Elizabeth Wulcan

Baum-Bott residue currents

Given a (singular) holomorphic foliation \mathcal{F} on a complex manifold, in the 70's Baum and Bott constructed residue classes associated with the singular components of \mathcal{F} . These Baum-Bott residues classes are locally defined (in the sense that they only depend on \mathcal{F} close to the singular component) and satisfy an index theorem which computes characteristic classes of \mathcal{F} . In the special case when \mathcal{F} has rank one and isolated singularities, they can be expressed in terms of the classical Grothendieck residue.

- 15:30-16:20

Jasmin Raissy

Spiralling domains in dimension 2

In this talk, I will present a joint work in progress with Xavier Buff. We study the dynamics of polynomial endomorphisms of \mathbb{C}^2 which are tangent to the identity at a fixed point. Our goal is to show the existence of such maps for which the immediate basin of attraction of the fixed point has an infinite number of distinct invariant connected components, where the orbits converge to the fixed point without being tangent to any direction.

- 16:30-17:20

Han Peters

Measure theoretic entropy of entire maps

The entropy of rational maps has been described in classical works of Misiurewicz and Przytycki, Gromov, Lyubich, Freire, Lopes and Mane, and Bedford, Lyubich and Smillie. The topological entropy of a rational function of degree d equals $\log(d)$, and the same statement holds for Hénon maps in two complex variables. Moreover, for maps in each of these classes there exists a unique measure of maximal entropy.

In an ongoing project with Leandro Arosio, Anna Miriam Benini and John Erik Fornæss, we consider the dynamics of so-called transcendental Hénon maps, combining ideas from transcendental dynamics in one variable and polynomial dynamics in two variables. It was suggested by Nessim Sibony and Romain Dujardin that the entropy of transcendental Hénon maps should always be infinite. Indeed, following the ideas of Marcus Wendt for entire maps in one variable, we previously showed that the topological entropy is always infinite.

In current work we consider the measure theoretic entropy of entire maps. We first construct several classes of entire transcendental functions for which there exist ergodic measures with infinite entropy, and then show that some of these constructions can be generalized to two complex variables. In all cases the support of the ergodic measures equals the respective Julia sets.

Friday, 30th June

- 10:00-10:50

Andrew Zimmer

A metric analogue of Hartogs' theorem

In this talk I will discuss a version of Hartogs' theorem where the holomorphic function is replaced by a locally symmetric Hermitian metric. As an application, I will explain how it implies the following rigidity result: if the Kobayashi metric on a strongly pseudoconvex domain is a Kähler metric, then the universal cover of the domain is biholomorphic to the unit ball. This is joint work with H. Gaussier.

- 11:30-12:20

Zbigniew Blocki

Rearrangements and the Monge–Ampère equations

Comparison results for rearrangements of solutions to elliptic equations are known for linear equations (Talenti) and for the real Monge–Ampère equation (Talenti, Tso). The main tool in the proof is always an appropriate isoperimetric inequality. These results often lead to optimal estimates, as they reduce the problem to radially symmetric solutions. We will discuss a similar question for the complex Monge–Ampère equation.