Compactness for continuous families of models

Kristóf Kanalas

A continuous family of models is a model internal to the category $Sh(X)$ of sheaves over a topological space X. More explicitly: it is a sheaf of L-structures, such that over each point the stalk is a model. I will give several examples, showing that an ultraproduct of models in $Sh(X)$ may not be a model. So compactness in general does not survive. But an instance of it does: we can still realize (continuous families of) types by (continuous families of) models, at least when X is the spectrum of a complete Boolean algebra. To get the same result for arbitrary spaces, it would be enough to prove it when X is the spectrum of a complete distributive lattice.