Countably compact vs pseudocompact spaces

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A topological space X is said to be *countably compact* iff every infinite subset has an accumulation point. A topological space X is *completely regular* if and only if it can be embedded as a subspace of a product of real lines $\mathbb{R}^{\kappa}$, where κ is a cardinal. A completely regular topological space X is *pseudocompact* iff every continuous function from X into $\mathbb{R}$ is bounded.

Clearly, a countably compact space is pseudocompact provided that it is completely regular. But what about the reverse implication? This seemingly innocent question has lead to the definition of wide array of topological properties and plethora of results describing the relationship between them.