Layer Algebra Decomposition: impacts on the structure theory of residuated lattices and substructural logics

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The layer algebra decomposition method, recently introduced in [5, 6], refines the well-established localization approach in semigroup theory, namely the process of localizing, for an idempotent u, to the universe of the greatest unitary subsemigroup of X with u as its unit. Notably, [6] demonstrates how the class of odd or even involutive residuated chains can be decomposed into a direct system of simpler algebras. These algebras can then be reconstructed into the original algebra using two key concepts: Plonka sums [10] and the directed lexicographic order introduced in [6].

This structural framework marks a significant departure from the traditional reliance on assumptions such as integrality, idempotency, or natural ordering, which have historically dominated the structural analysis of residuated lattices. The layer algebra decomposition method has already proven to be transformative, facilitating structural descriptions for various classes of residuated lattices, as detailed in [1, 4, 2, 3, 7, 8, 9, 11].

In this talk, we will summarize the ongoing research in this area and highlight the achievements realized thus far.

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