Second Analysis Mathematica Conference

Conference Booklet

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The Second Analysis Mathematica Conference takes place in Budapest, Hungary between July 29 and August 2, 2024.

This event is to promote the 50-years-old journal Analysis Mathematica and provides a good opportunity for professors and young researchers, current or future authors of Analysis Mathematica to present their results and to foster possible collaborations.

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Factorization Results and Estimations via a Generalized Taylor Theorem

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This work introduces the generalized Taylor theorem for constant coefficient linear differential operators using divided differences with repeated arguments [AP22]. It extends to exponential polynomials, includes integral remainder terms and mean value theorems, and presents factorization results and estimates for linear functionals. Error bounds applicable to the trapezoidal and Simpson rules are also analyzed [AP24].

References

[AP22] Ali Hasan Ali and Zsolt Páles. Taylor-type expansions in terms of exponential polynomials. *Math. Inequal. Appl.*, 25(4):1123–1141, 2022. ISSN 1331-4343,1848-9966. URL https://doi.org/10.7153/mia-2022-25-69.

[AP24] Ali Hasan Ali and Zsolt Páles. Estimates of linear expressions through factorization. *J. Approx. Theory*, 299:Paper No. 106019, 23, 2024. ISSN 0021-9045,1096-0430. URL https://doi.org/10.1016/j.jat.2024.106019.

Properties of Whittaker functions with applications

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In this talk we present some new properties of Whittaker functions of the first and the second kind. Some well-known results of Grönwall on logarithmic derivative of modified Bessel functions of the first kind concerning exponential bounds are extended to Whittaker functions of the first and second kind $M_{\kappa,\mu}$ and $W_{\kappa,\mu}$. Moreover, a complete monotonicity result is proved for the log-arithmic derivative of the Whittaker function $W_{\kappa,\mu}$, and some monotonicity results with respect to the parameters and argument are shown for the log-arithmic derivative of $M_{\kappa,\mu}$. The results extend and complement the known results in the literature about modified Bessel functions of the first and second kind. In addition by using an integral representation of Ismail and Kelker for the quotient of Tricomi hypergeometric functions, the infinite divisibility and self-decomposability of the recently defined four parameter lifetime Whittaker distribution is investigated, which is a natural extension of the classical gamma, exponential, chi-square, generalized Lindley, Lindley, beta prime, and Lomax distributions. The Whittaker distribution also belongs to the class of hyperbolically completely monotone distributions and generalized gamma convolutions, and it is a super-Gaussian distribution. By using some results for the moments of the Whittaker distribution some Turán type inequalities are deduced for the Whittaker functions of the second kind and as an application it is proved that the effective variance of the Whittaker distribution is bounded from below. The talk is based on the following papers:

References

[AB23a] Genet M. Assefa and Árpád Baricz. Exponential bounds for the logarithmic derivative of Whittaker functions. *Proc. Amer. Math. Soc.*, 151(11):4867–4880, 2023. ISSN 0002-9939,1088-6826. URL `https://doi.org/10.1090/proc/16549`.

[AB23b] Genet M. Assefa and Árpád Baricz. Infinite divisibility of the Whittaker distribution. *Proc. Amer. Math. Soc.*, 151(12):5429–5442, 2023. ISSN 0002-9939,1088-6826. URL `https://doi.org/10.1090/proc/16562`.

Around Furstenberg's Times 2, Times 3 Conjecture: A Survey of Conjectures and Results

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In this talk, we explore Furstenberg's times-2, times-3 conjecture, which questions whether the normalized Lebesgue measure on the circle is the only atom-free probability measure invariant under both the times-2 and times-3 transformations. We survey several related results and conjectures, providing a comprehensive overview of the topic. The original results presented in this talk are joint work with Sophie Grivaux.

Lipschitz images and dimensions

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We consider the question which compact metric spaces can be obtained as a Lipschitz image of the middle third Cantor set, or more generally, as a Lipschitz image of a subset of a given compact metric space.

In the general case we prove that if A and B are compact metric spaces and the Hausdorff dimension of A is bigger than the upper box dimension of B, then there exist a compact set $A' \subset A$ and a Lipschitz onto map $f\colon A' \to B$.

As a corollary we prove that any 'natural' dimension in $\mathbb{R}^n$ must be between the Hausdorff and upper box dimensions.

We show that if A and B are self-similar sets with the strong separation condition with equal Hausdorff dimension and A is homogeneous, then A can be mapped onto B by a Lipschitz map if and only if A and B are bilipschitz equivalent.

For given $\alpha > 0$ we also give a characterization of those compact metric spaces that can be obtained as an α-Hölder image of a compact subset of $\mathbb{R}$. The quantity we introduce for this turns out to be closely related to the upper box dimension.

This is a joint work with Tamás Keleti.

On L_p Markov factor of cuspidal domains in $\mathbb{R}^d$

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We denote by $\mathcal{P}_n(\mathbb{R}^d)$ the space of all polynomials of d real variables with real coefficients of degree at most n. Let $K \subset \mathbb{R}^d$ be a compact set with non-empty interior. The nth order Markov factor of K is defined by

$$M_{n,p}(K) := \sup \left\{ \frac{\|D^\alpha P\|_{L_p(K)}}{\|P\|_{L_p(K)}} : \ P \in \mathcal{P}_n(\mathbb{R}^d), P \not\equiv 0, |\alpha| = 1 \right\}.$$

The purpose of this talk is to study the magnitude of $M_{n,p}$ for a certain family of domains with cusps. The considered areas also include those that do not satisfy Markov's inequality.

Orthogonal polynomials and the associated Jacobi operator

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To a sequence of orthonormal polynomials p_n on the real line is associated a Jacobi operator $(T, D(T))$, i.e., the operator in ℓ^2 defined as the closure of the Jacobi matrix acting on the subspace of complex sequences with only finitely many non-zero terms. It is well-known that it is symmetric with deficiency indices either $(0,0)$ or $(1,1)$. The two cases correspond to the moment problem behind being either determinate or indeterminate, i.e., there is exactly one orthogonality measure or there are several and then infinitely many orthogonality measures for p_n. In the determinate case $(T, D(T))$ is self-adjoint, but not in the indeterminate case, where it has infinitely many self-adjoint extensions. We shall focus on this case, and for a complex number z we let $\mathfrak{p}_z, \mathfrak{q}_z$ denote the sequences $(p_n(z))$ and $(q_n(z))$, where q_n denote the polynomials of the second kind. These sequence are known to be square summable.

It is known that $\mathfrak{p}_z, \mathfrak{q}_z \notin D(T)$ for all $z \in \mathbb{C}$. We determine whether linear combinations of $\mathfrak{p}_u, \mathfrak{p}_v, \mathfrak{q}_u, \mathfrak{q}_v$ for $u, v \in \mathbb{C}$ belong to $D(T)$ or to the domain of the self-adjoint extensions of T in ℓ^2. The results depend on the four Nevanlinna functions of two variables associated with the moment problem. We also show that $D(T)$ is the common range of an explicitly constructed family of bounded operators on ℓ^2.

The talk is based on recent joint work with Ryszard Szwarc, Wrocław, see arXiv:2301.00586.

Universal bounds for energy of weighted spherical codes and designs

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We obtain universal lower and upper bounds for the energy of weighted spherical codes of fixed dimension, cardinality, and weights. The universality is of the sense of Levenshtein (there is a bound for every code), in the sense of Cohn-Kumar (attaining codes, if any, are universally optimal), and in terms of main parameters (nodes and weights are independent on the potentials). Further, our bounds are valid for a large class of potentials that includes absolutely monotone functions of inner products. Bounds are also obtained for the energy of weighted spherical designs. We explore our bounds for several previously studied weighted spherical codes.

This is a joint work with S. Borodachov, P. Dragnev, D. Hardin, E. Saff, M. Stoyanova.

Bohr's theorem for Beurling integer systems

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Bohr's theorem is a theorem about (ordinary) Dirichlet series; it says that if $f(s) = \sum_n a_n n^{-s}$ converges somewhere and has bounded analytic extension to $\Re s > 0$, then the series converges uniformly for $\Re s > \varepsilon$, for every $\varepsilon > 0$. For general Dirichlet series $\sum_n a_n e^{-\lambda_n s}$ Bohr's theorem need not hold. Nonetheless, having this theorem is desirable; it is for example a necessity for having a satisfactory Hardy space theory of Dirichlet series.

In this talk we first give some introduction on the theory of general Dirichlet series and the associated Hardy spaces on the one hand, and on so-called Beurling generalized number systems on the other hand. Such systems consist of increasing real sequences of *generalized primes* $\mathcal{P} = (p_1, p_2, \dots)$ and *generalized integers* $\mathcal{N} = (n_1, n_2, \dots)$ which are formed by taking all possible products of the generalized primes. With each such number system one can associate the class of general Dirichlet series $\sum_k a_k e^{-\lambda_k s} = \sum_k a_k n_k^{-s}$.

In the second part of the talk, we show that there exist Beurling number systems $(\mathcal{P}, \mathcal{N})$ for which Bohr's theorem holds for the class of associated Dirichlet series, and additionally, for which the Riemann hypothesis holds. This leads to a counterexample to a conjecture of Helson on outer functions in Hardy spaces of general Dirichlet series.

This talk is based on joint work with A. Kouroupis and K.-M. Perfekt from NTNU (Norway).

Takagi Dynamics

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After mentioning some earlier results concerning Takagi's function we discuss some recent ones related to the dynamics of this function.

The talk is mainly based on a joint project with Jesús Llorente Jorge.

On the boundary behavior of bounded analytic functions

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We prove a result which sheds a new light on the fundamental theorems of complex analysis due to P. Fatou, F. and M. Riesz, N.N. Lusin, I.I. Privalov, and A. Beurling. Only classical tools available at the times of Fatou are used. The proofs are very simple and short.

Also, we present the solution to an open problem proposed by M. von Renteln in 1980, which has been published in a well-known collection of open problems by D.M. Campbell, J.G. Clunie, and W.K. Hayman.

Convergence, almost convergence of triple sequences and their interconnection in the environment of uncertainty

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Convergence concept of triple sequences is introduced in all five directions of uncertainty. Although the relationships between different convergence concepts behaves similarly like the case of single sequence, we have established few more new interconnections which completes the purpose to some extent. Furthermore, we have presented the interrelationships between convergent, bounded and almost convergent complex uncertain triple sequences in a given uncertainty space.

Quantified versions of the Ingham-Karamata theorem

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The classical Ingham-Karamata Tauberian theorem states the following.

Let $\rho \in L^\infty(0,\infty)$. If $\mathcal{L}\{\rho;s\}/s$, initially convergent on $\operatorname{Re} s > 0$ admits a continuous extension to the line $\operatorname{Re} s = 0$, then

$$\int_0^x \rho(u)\mathrm{d}u = o(1), \quad x \to \infty. \tag{1}$$

This theorem has plenty of applications within mathematical analysis, varying from the prime number theorem to obtaining rates of decay for solutions of partial differential equations. It is considered to be one of the most influential Tauberian theorems of the last century.

We will discuss recently obtained quantified versions of the Ingham-Karamata theorem under various assumptions for the Laplace transform and indicate how to prove their optimality. Moreover, instead of the two-sided Tauberian condition that $\rho \in L^\infty(0,\infty)$, we shall work under flexible one-sided Tauberian conditions and mention how this is advantageous for certain applications.

Optimal polarization (PULB) pairs of codes found in the Leech lattice

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It was previously shown by the authors that the discrete potentials of almost all known sharp codes attain universal lower bounds for polarization (PULB) for spherical τ-designs, where "universal" is meant in the sense of applying to a large class of potentials that includes absolutely monotone functions of inner products and in the sense that the computational parameters of the bound are invariant with respect to the potential. In this talk we characterize the sets of universal minima D for some of these sharp codes C found in the Leech lattice and establish a duality relationship, namely that the normalized discrete potentials of C and D have the same minimum value and the sets C and D are each others minima sets (up to antipodal symmetrization in some cases). The extremal duality is obtained by utilizing the natural embedding of the PULB pair codes in the Leech lattice and its properties, which simplifies the analysis significantly. In the process we discover a new universally optimal code in $\mathbb{RP}^{21}$ with 1408 points.

This is a joint work with S. Borodachov, P. Boyvalenkov, D. Hardin, E. Saff, and M. Stoyanova.

Fourier Quasicrystals, crystalline measures, and their generalisations

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A complex discrete measure on the Euclidean space with locally finite support and temperate growth is called crystalline if its Fourier transform is also a measure with locally finite support. If the variations of both measures are temperate distributions, then the measure is called a Fourier Quasicrystal.

We obtain some sufficient conditions for a crystalline measure to be a Fourier Quasicrystal, and present an example of a crystalline measure that is not a Fourier Quasicrystal. Also, we consider positive temperate measures with locally finite support, for which the Fourier transform is a pure point measure. We present some properties of these measures, in particular various uniqueness theorems. In the case of dimension 1 we obtain necessary and sufficient conditions for the support of the measures with integer masses to be the zero set of an absolutely convergent Dirichlet series with bounded spectrum and real zeros.

Generalized Mittag-Leffler-Confluent Hypergeometric Functions in Fractional Calculus with Integral Operator

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Originally, confluent hypergeometric functions and Mittag-Leffler functions were developed to approximate interpolation in the exponential function. The purpose of this work is to investigate the operators of generalized Mittag-Leffler-type functions. Under certain unusual circumstances, the generalized Fox-Wright function will be utilized. Furthermore, we will investigate some of the commonly employed generalized fractional integral operators in fractional calculus. Researchers have recently learned more about generalized Mittag-Leffler-type functions by using generalized fractional differential and integral operators. The study's results add to this wealth of knowledge. There will also be a thorough analysis of the various implications and effects of these results.

Turán type oscillation inequalities in L^q norm on the boundary of convex domains

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In 1939 Pál Turán and János Erőd initiated the study of lower estimations of maximum norms of derivatives of polynomials, in terms of the maximum norms of the polynomials themselves, on convex domains of the complex plane. As a matter of normalization they considered the family $\mathcal{P}_n(K)$ of degree n polynomials with all zeros lying in the given convex, compact subset $K \Subset \mathbb{C}$. Turán obtained the first results for the interval $\mathbb{I} := [-1, 1]$ and the disk $\mathbb{D} := \{z \in \mathbb{C} \, : \, |z| \leq 1\}$. The order of the optimal constant was found to be $\sqrt{n}$ for $\mathbb{I}$ and n for $\mathbb{D}$. Erőd extended investigations to other compact convex domains, too. It took until 2006 to clarify that all compact convex *domains* (with nonempty interior), follow the pattern of the disk, and admit an order n inequality.

For $L^q(\partial K)$ norms with any $1 \leq q < \infty$ we obtained order n results for various classes of domains. Further, in the generality of all convex, compact domains we could show a $cn/\log n$ lower bound together with an $O(n)$ upper bound for the optimal constant. Also, we conjectured that all compact convex domains admit an order n Turán type inequality. Now we prove this for all *polygonal* convex domains and any $0 < q < \infty$.

Joint work with Szilárd Révész.

Linear topological structure of the spaces of Whitney jets

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We discuss some recent results on Whitney jet spaces defined on the sequence of points $K = (a_k)_{k=0}^{\infty} \cup \{0\}$. We first present a basis in Whitney space $\mathcal{E}(K)$ under the condition of a moderate rate of convergence of the sequence. We then present a geometric criterion for the existence of a projection in the space $C^{\infty}[0, 1]$, that can be factorized through the constraint space $C_{\infty}(K)$. Finally, we consider infinite products of spaces of power series of infinite type associated with the space $\mathcal{E}(K)$.

Evolution equations with monotone operators in Hilbert spaces and applications

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Existence, uniqueness and stability of solutions to evolution equations in Hilbert spaces are investigated. The results are applied to prove theorems on solvability of second order parabolic stochastic partial differential equations with singular lower order coefficients. The talk is based on a recent joint work with N.V. Krylov at the University of Minnesota.

The prime geodesic theorem in arithmetic progressions

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One can count hyperbolic conjugacy classes in $\mathrm{SL}_2(\mathbb{Z})$ according to their traces. The result is the prime geodesic theorem, which bears a close similarity with the prime number theorem. As primes are equidistributed in reduced residue classes, the natural question arises if the same is true of the traces mentioned above. It turns out that the answer is no, and the corresponding non-uniform distribution can be determined explicitly. This confirms a conjecture of Golovchanskiĭ–Smotrov (1999).

Based on joint work with Dimitrios Chatzakos and Ikuya Kaneko.

Estimation of function's supports under arithmetic constraints

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The well-known inequality $|\operatorname{supp}(f)||\operatorname{supp}(\widehat{f})| \geq |G|$ gives a lower estimation for each support. In the present talk we consider the case where there exists a slowly increasing function F such that $|\operatorname{supp}(f)| \leq F(|\operatorname{supp}(\widehat{f}|))$. We will show that this can be done under some arithmetic constraint. The two links that help us come from additive combinatorics and theoretical computer science. The first is additive energy which plays a central role in additive combinatorics. The second the influence of Boolean functions. Our main tool is the spectral analysis of Boolean functions. We prove an uncertainty inequality in which the influence of a function and its Fourier spectrum play a role.

Trace inequality with Bessel convolution

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Considering potentials defined by Bessel kernel with Bessel convolution a Kerman–Sawyer type characterization of trace inequality is given. As an application an estimate on the least eigenvalue of Schrödinger–Bessel operators is derived. This is joint work with Mouna Chegaar.

Mourre inequality for non-local Schrödinger operators

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We consider the Mourre inequality for the following self-adjoint operator $H = \Psi(-\Delta/2) + V$ acting on $L^2(\mathbb{R}^d)$, where $\Psi : [0, \infty) \to \mathbb{R}$ is an increasing function, Δ is Laplacian and $V : \mathbb{R}^d \to \mathbb{R}$ is an interaction potential. Mourre inequality immediately yields the isolateness and finite multiplicity of the eigenvalues. Moreover, Mourre inequality has the application to the absence of the singular continuous spectrum by combining the limiting absorption principle and, in addition, Mourre inequality is also used for proof of the minimal velocity estimate that plays an important role in the scattering theory. In this talk, we report one of the results in [ILS22] that Mourre inequality holds under the general Ψ and V by choosing the conjugate operator $A = (p \cdot x + x \cdot p)/2$ with $p = -\sqrt{-1}\nabla$, and that the isolateness and finite multiplicity of the eigenvalues hold.

References

[ILS22] Atsuhide Ishida, József Lőrinczi, and Itaru Sasaki. Absence of embedded eigenvalues for non-local Schrödinger operators. *J. Evol. Equ.*, 22(4):Paper No. 82, 30, 2022. ISSN 1424-3199,1424-3202. URL `https://doi.org/10.1007/s00028-022-00836-0`.

Value distribution theory and complex difference equations

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In this talk, we discuss linear and nonlinear difference equations in the complex plane. First we back to the works of existence theorems of merompprphic solutions due to Nörlund, Milne-Thomson, and Whittaker in the early 20th century. Next we back the researches of complex difference equations by means of Nevanlinna theory in 1970's and 1980's due to Yanagihara and Bank. Finally, we mention some recent results by the difference analogues of Nevanlinna theory and Wiman-Valiron theory after 2000.

Constructing chaotic maps

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There are many notions aiming at a formal definition of chaos on a topological dynamical system, yet three most widely recognized are Devaney chaos, Li-Yorke chaos and positive topological entropy. Consequently the LEO (locally eventually onto) maps with dense set of periodic points are of interest, as such maps are chaotic with respect to all the three notions mentioned above. For arbitrary Peano continuum we describe a construction of such a selfmap. We further investigate various sets of maps with some of the "chaotic" properties.

The work is in progress.

Banach Algebras of Convolution Type Operators with Oscillating Data on Weighted Lebesgue Spaces

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Let $\mathcal{B}_{p,w}$ be the Banach algebra of all bounded linear operators acting on the weighted Lebesgue space $L^p(\mathbb{R}, w)$, and let $\mathcal{K}_{p,w}$ be the ideal of all compact operators in $\mathcal{B}_{p,w}$, where $p \in (1, \infty)$ and w is a Muckenhoupt weight. The Banach algebra $\mathfrak{A}_{p,w} \subset \mathcal{B}_{p,w}$ generated by all multiplication operators aI with $a \in PQC$ and by all convolution operators $W^0(b)$ with $b \in PSO^{\diamond}_{p,w}$ is studied, where $PQC \subset L^\infty(\mathbb{R})$ is the C^*-algebra of piecewise quasicontinuous functions, $PSO^{\diamond}_{p,w} \subset M_{p,w}$ is the Banach algebra of piecewise slowly oscillating functions that admit piecewise slowly oscillating discontinuities at arbitrary points of $\mathbb{R} \cup \{\infty\}$, and $M_{p,w}$ is the Banach algebra of Fourier multipliers on $L^p(\mathbb{R}, w)$. For any $p \in (1, \infty)$ and any Muckenhoupt weight w, we study the Fredholmness of operators in the Banach algebra $\mathcal{Z}_{p,w} \subset \mathfrak{A}_{p,w}$ generated by the operators $aW^0(b)$ with quasicontinuous functions $a \in QC$ and slowly oscillating functions $b \in SO^{\diamond}_{p,w}$. Describing the maximal ideal space of the commutative quotient Banach algebra $\mathcal{Z}^{\pi}_{p,w} := \mathcal{Z}_{p,w}/\mathcal{K}_{p,w}$ being a central subalgebra of the quotient Banach algebra $\mathfrak{A}^{\pi}_{p,w} := \mathfrak{A}_{p,w}/\mathcal{K}_{p,w}$, making use of the limit operators techniques and applying the Allan-Douglas local principle with the two idempotents theorem, we construct a Fredholm symbol calculus for the Banach algebra $\mathfrak{A}_{p,w}$ and establish Fredholm criteria for the operators $A \in \mathfrak{A}_{p,w}$.

On some classes of linear relations

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Linear relations, particularly in their essential and semi-regular forms, constitute a rich field of research with significant theoretical and practical implications.

In this presentation, I will share the findings of my work on the structure and properties of linear relations, emphasizing essentiality, semi-regularity and improjectivity. My research covers several key aspects, detailed in the following studies:
I will introduce some basic classes of linear relations and define new classes such as essentially semi-regular and improjective linear relations. I will present a decomposition theorem of Kato for essentially semi-regular linear relations and discuss their matrix structures.

An important area where improjective linear relations find natural application is the indecomposability of Banach spaces. I will provide a characterization of Banach spaces in terms of improjective linear relations, demonstrating how these concepts can offer new insights and solutions to longstanding problems in functional analysis.

In conclusion, this presentation will provide an overview of recent advances in the study of linear relations, focusing on their structures, characterizations, and the interactions between different class es of relations. Attendees will learn how these research findings can be applied to improve methods for solving linear systems and address complex problems in various scientific and engineering domains.

Fuglede's conjecture on finite Abelian groups and finite geometry

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In this talk we introduce Fuglede's conjecture on finite Abelian groups and summarize the most recent developments. We discuss the main techniques that are based on equidistributivity property and its generalizations. In some special case this naturally lead to the study of number of special directions in $\mathbb{Z}_p^2$, which is connected to some classical results of Rédei-Megyesi and Lovász-Schrijver. With our concept we reproved these results using projection polynomials and discrete harmonic analysis.

Extension property

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I shall discuss my last paper with John McCarthy on extension property for the polydisc.

A second-order Magnus-type integrator for evolution equations with delay

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Let us consider the equation $\frac{\mathrm{d}}{\mathrm{d}t}Y(t) = A(t)Y(t)$ $(t \geq 0)$, $Y(0) = Y_0$, where $Y(t)$ and $A(t)$ are linear operators on appropriate spaces. In the case when all of the operators $A(t)$ commute, the solution takes the simple form $Y(t) = \exp\left(\int_0^t A(s)\,\mathrm{d}s\right) Y_0$. In the general, noncommutative case, however, one has to correct the exponent, and the exact solution is given by the Magnus series expansion, involving integrals of commutators. The first term of this expansion corresponds to the commutative solution, and is a good approximant leading to a second-order numerical method using the midpoint rule to approximate the integral in the exponent:

$$\widehat{Y}_{n+1}^{(\tau)} = \mathrm{e}^{\tau A((n+1/2)\tau)}\widehat{Y}_n^{(\tau)}, \quad n \in \mathbb{N}$$

where $\tau > 0$ is an arbitrary timestep and $\widehat{Y}_n^{(\tau)}$ denotes the numerical approximation to $Y(n\tau)$ with initial value $Y_0^{(\tau)} := Y_0$.

We are interested in problems of the form

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}u(t) = Q(F({}_t u))u(t), & t \in [0,\infty), \\ u(s) = \varphi(s) \in X, & s \in [-\delta, 0], \end{cases}$$

where X is a Banach space, ${}_t u\colon [-\delta, 0] \to X$ denotes the δ-history ${}_t u(s) := u(t+s)$ of the solution at time t (i.e., the initial condition could also be written as ${}_0 u = \varphi$ with some initial history function $\varphi\colon [-\delta, 0] \to X$), $Q(w) = Q_0 + \widetilde{Q}(w)$ where Q_0 is an unbounded operator on X and $\widetilde{Q}(w)$ is bounded for all $w \in X$, and $F \in \mathrm{C}([-\delta, 0], X)$ actually only depends on the restriction to $[-\delta, -\epsilon]$ for some fixed $\epsilon \in (0, \delta]$.

We rewrite this abstract delay equation as a nonautonomous abstract Cauchy problem allowing us to introduce a Magnus-type integrator for the former. We prove the second-order convergence of the obtained Magnus-type integrator. We also show that if the differential operators involved admit a common invariant set for their generated semigroups, then the Magnus-type integrator will respect this invariant set as well, allowing for much weaker assumptions to obtain the desired convergence. As an illustrative example we consider a space-dependent epidemic model with latent period and diffusion.

Joint work with Petra Csomós.

The discrete Pompeiu problem

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We say that a finite subset E of the Euclidean space $\mathbb{R}^k$ has the discrete Pompeiu property (dPp) if, whenever $f\colon \mathbb{R}^k \to \mathbb{C}$ is such that the sum of the values of f on any congruent copy of E is zero, then f is identically zero. t is easy to see that every set having at most three elements has dPp. Applying harmonic analysis in some varieties connected to the problem and also some results of Euclidean Ramsey theory, it was proved by Cs. Vincze and the present authors in 2018 that, for $k = 2$, every parallelogram and every quadrangle with rational coordinates has dPp. In the talk we prove, by improving upon the previous methods and using also results on linear equations of units, that every finite subset of $\mathbb{R}^k$ has dPp for every k. We also discuss the connections with the discrete Steinhaus tiling problem. The problem is to decide, for a given set $K \subset \mathbb{R}^k$ if there is a set S that intersects every congruent copy of K in exactly one point. A finite set $K \subset \mathbb{R}^k$ is called a *Jackson set* if there is no such set S. It has been proved that every set of cardinality $2, 3, 4, 5$ or 7 is a Jackson set. We show, applying the result on the dPd property, that every finite subset $\mathbb{R}^k$ of cardinality at least two is a Jackson set.

Joint work with Gergely Kiss.

Mapping spaces and holomorphic functions

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Continuous maps from a fixed compact Hausdorff space to a finite dimensional complex manifold themselves form a complex manifold, typically infinite dimensional. We will discuss two theorems about holomorphic functions on such mapping spaces, one analogous to the Monodromy theorem (but with a twist), the other to Liouville's theorem (also with a twist).

Establishing Wavelet Series Expansion via Approximate Duals

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Characterizing complete wavelet systems within various function spaces has been a foundational research subject for decades. Traditionally, numerous studies have aimed to identify sufficient conditions on mother wavelets to ensure that the resulting wavelet systems form exact duals within a given function space. More recently, approximate duals, a generalized form of exact duals, have been introduced, allowing a wavelet series expansion for functions within Lebesgue spaces $L^p(\mathbb{R})$ and the Hardy space $H^1(\mathbb{R})$. In this talk, we provide sufficient conditions for mother wavelets to form approximate duals across the entire range of Hardy spaces $H^p(\mathbb{R})$, where $0 < p \leq 1$. Building upon these conditions, we present a wavelet series expansion for functions within Hardy spaces. The key features of our approach include the following: (i) our results work for any $0 < p \leq 1$; (ii) we do not assume that the mother wavelets are exact duals in $L^2(\mathbb{R})$; and (iii) we provide a tractable bound for the operator norm of the associated wavelet frame operator so that it is possible to check the suitability of mother wavelets.

A characterization of the nowhere differentiable functions in the Generalized Takagi class

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In this talk, we will explore the non-differentiability behavior of the functions belonging to the Generalized Takagi class, a recent generalization of the Takagi function. It is derived by replacing the dyadic numbers used in the definition of classical Takagi function with an arbitrary countable and dense subset of $[0, 1]$. Subject to specific conditions on the decomposition of such a set, we will show that a function within this class is nowhere differentiable if and only if the sequence of weights does not belong to c_0.

On the size of the set of Riemann but not Darboux integrable functions

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We say that a subset M of a Banach space is lineable if $M \cup \{0\}$ contains an infinite-dimensional vector subspace, dense-lineable if $M \cup \{0\}$ contains an infinite-dimensional dense vector subspace. In this note we derive lineability of set of Riemann integrable functions which are not Darboux integrable for some Banach spaces.

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Spectral properties of a class of non-local Schrödinger operators

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A non-local Schrödinger operator is obtained as a sum of a non-local operator and a multiplication operator. Typical examples of a non-local operator are fractional and relativistic Laplacians, which are generators of Lévy processes. The perturbation by a multiplication operator may give rise to a discrete component in the spectrum. We will discuss some problems related to the eigenvalues and eigenfunctions of the so obtained non-local Schrödinger operators.

Embedding distance graphs into thin sets of Euclidean spaces

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A distance graph Γ is a graph embedded into a Euclidean space. We say two such graphs Γ and Γ' are isometric if there is a bijection between their vertices which keeps the lengths of the corresponding edges, and similar if one is isometric to a dilate of the other.

We prove that for all $k \geq 2$, and for all distance graphs $\Gamma \subseteq \mathbb{R}^k$ of degree at most $k-1$ the following holds;
There is a dimension $s < k$ such that any compact set $E \subseteq \mathbb{R}^k$ of Hausdorff dimension $dim(E) > s$ contains an isometric copy of the dilates $\lambda \cdot \Gamma$, for all $\lambda \in I$ for some non-trivial interval I.

In particular, such sets E contain a regular k-simplex Δ_k of side-length λ, for all $\lambda \in I$, which extends a result of Bourgain about embedding of similar copies of k-simplices into sets $E \subseteq \mathbb{R}^k$ of positive measure or density. We will also highlight some related open problems.

Fourier analytic bounds on the density of 1-avoiding sets

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We prove a conjecture of Erdős that the upper density of any measurable planar set that avoids the unit distance is less than 1/4. The proof uses Fourier analysis and linear programming.

Joint work with G. Ambrus, A. Csiszárik, D. Varga and P. Zsámboki.

On the connectivity of graph Lipscomb's space

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A central role in topological dimension theory is played by Lipscomb's space J_A since it is a universal space for weight $|A| \geq \aleph_0$ metric spaces. On the one hand, Lipscomb's space is the attractor of a possibly infinite iterated function system, i.e., it is a generalized Hutchinson-Barnsley fractal. As, on the other hand, some classical fractal sets are universal spaces, one can conclude that there exists a strong connection between topological dimension theory and fractal set theory.

Recently a generalization of Lipscomb's space was introduced (see [MM22]). It is denoted by $J_A^{\mathcal{G}}$ and it is called graph Lipscomb's space associated with the graph G on the set A. It turned out that it is a topological copy of a generalized Hutchinson-Barnsley fractal. We provide a characterization of those graphs G for which $J_A^{\mathcal{G}}$ is connected. In the particular case when A is finite, we give some supplementary characterizations.

Joint work with Wieslaw Kubiś, Alexandru Mihail, and Magdalena Nowak.

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Some Equivalence Theorems for Monotone Quasilinear Operators

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In this study, some equivalence theorems for the the boundedness of the composition of a quasilinear operator with the Hardy and Copson operators in weighted Lebesgue spaces are presented. The usefulness of the obtained results is illustrated in the case of weighted Hardy-type and weighted iterated Hardy-type inequalities.

Join work with Merve Görgülü.

Minimax problems for finite sums of kernel functions

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Bojanov, Uluchev and others investigated minimax problems for Blaschke products assuming all the zeros are in the same interval and the multiplicities are fixed. After taking logarithm, we obtain sums of the same kernel function of the form $F(x,t) = J(t) + \sum_{j=1}^{n} \nu_j K(x_j, t)$ where $J(t)$ is a concave, continuous function on $[-1,1]$, $\nu_j > 0$, $\mathbf{x} = (x_1, \ldots, x_n)$, $-1 \leq x_1 \leq \ldots \leq x_n \leq 1$ and $K(x,t)$ is a kernel function.

In this setting we assume different properties of the kernel function, and we investigate the existence and the behavior of maxima, we establish perturbation lemma type results and show homeomorphism for differences of maxima.

This is a work in progress, joint with Szilárd Révész.

Some properties of Nörlund means of Walsh-Fourier series

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In the presented talk we discuss the Nörlund means of Walsh-Fourier series with monotone generating sequences. In particular, we discuss the norm convergence of Nörlund means in L_1 space and in C_W space in terms of modulus of continuity [MS92, Nag12, Nag10, GN23]. The results are related to the corresponding Lebesgue constants of Nörlund means. In the paper [GN23], we proved two-sides estimates for Lebesgue constants of Nörlund means. Moreover, we discuss the almost everywhere convergence of Nörlund means under some assumption [GN23].

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Minimax and maximin problems for sums of translates on the real axis

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Sums of translates generalize logarithms of weighted algebraic polynomials. The talk presents the solution to the minimax and maximin problems on the real axis for sums of translates. We prove that there is a unique function that is extremal in both problems. The key in our proof is a reduction to the problem on a segment. For this, we work out an analogue of the Mhaskar-Rakhmanov-Saff theorem, too.

Jackson-type inequality in Hilbert spaces and on homogeneous manifolds

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We consider a Hilbert space $\mathbf{H}$ equipped with a set of strongly continuous bounded semi-groups satisfying certain conditions. The conditions allow to define a family of moduli of continuity $\Omega^r(s,f)$, $r \in \mathbb{N}$, $s>0$, of vectors in $\mathbf{H}$ and a family of Paley-Wiener subspaces PW_σ parametrized by bandwidth $\sigma>0$. These subspaces are explored to introduce notion of the best approximation $\mathcal{E}(\sigma,f)$ of a general vector in $\mathbf{H}$ by Paley-Wiener vectors of a certain bandwidth $\sigma>0$. The main objective of the paper is to prove the so-called Jackson-type estimate $\mathcal{E}(\sigma,f) \leq C\left(\Omega^r(\sigma^{-1},f)+\sigma^{-r}\|f\|\right)$ for $\sigma>1$. Our assumptions are satisfied for a strongly continuous unitary representation of a Lie group G in a Hilbert space $\mathbf{H}$. It allows to obtain the Jackson-type estimates on homogeneous manifolds.

Zero density theorems for analytic functions in Number Theory

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Zero-density theorems play an important role in the theory of Riemann, Dedekind and Beurling zeta functions and Dirichlet and other L-functions. They have a theoretical significance since they show, for example, that if the Riemann Hypothesis is not true then still the majority of zeros are on or near the critical line $\Re(s) = 1/2$. On the other hand, they have a great significance in arithmetical applications since they can substitute the Riemann or Generalized Riemann conjecture in many applications (like the Bombieri-Vinogradov theorem) or yield estimates that are nearly as strong as the consequences of the Riemann conjecture (like upper bounds on the difference of primes). In the lecture we present a general density theorem and consequences for Riemann, Dedekind and Beurling zeta functions.

Kolmogorov widths and entropy numbers of the classes of periodic functions in the space $B_{q,1}$

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We present the obtained exact-order estimates for the Kolmogorov widths and entropy numbers for the Nikol'skii-Besov classes $B^r_{p,\theta}$ of one and several variables in the space $B_{q,1}$ with $1 < q < \infty$, see [PR24].

It was shown, that for most of the relations between the parameters p, q, θ in the multivariate case ($d \geq 2$), in contrast to the univariate, the estimates in $B_{q,1}$ differ in order from the corresponding estimates of the mentioned approximation characteristics in the Lebesque space L_q, the norm in which is weaker than the corresponding $B_{q,1}$-norm.

Besides, a significant progress is made in respective estimates comparing to the known estimates in the space L_q.

Similar estimates for the Sobolev classes $W^r_{p,\alpha}$ were obtained in [PRR24].

Joint work with A.S. Romanyuk.

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On the existence of an extremal function for the Delsarte extremal problem

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In the general setting of a locally compact Abelian group G with identity 0, the Delsarte extremal problem asks for the supremum of integrals over the collection of continuous positive definite functions $f : G \to \mathbb{R}$ satisfying $f(0) = 1$ and having $\operatorname{supp} f_+ \subset \Omega$ for some measurable subset Ω of G of finite measure. We call this the Delsarte constant for the group G and subset Ω. When $G = \mathbb{R}^d$ and Ω is a ball in $\mathbb{R}^d$, the Delsarte constant gives an upper bound for the sphere packing density. Apart from computing the Delsarte constant, one can ask questions about the nature of an extremal function, if one exists, for a specific group G and a specific subset $\Omega \subset G$. In this talk, we consider the Delsarte problem in the setting of locally compact Abelian groups, and investigate the question of the existence of an extremal function.

c-removable sets

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We say a closed set F in a Euclidean space is c-removable if any continuous function that is locally convex on the complement of F is necessarily convex on the whole space. I would like to show some basic ideas regarding this notion as well as formulate several (possibly quite surprising) results about it. In a joint work with D. Pokorný and D. Hruška (which is now being prepared for publication) we have proved, among other results that I shall present, that there exist nontrivial continua that are c-removable, as well as (very small) non-c-removable copies of the Cantor set.

Extremal problems for algebraic polynomials under constraints on their zeroes

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We discuss several extremal problems on the set $P_n(G)$ of algebraic polynomials of degree n that do not vanish on an open set G. We study the Chebyshev problem on polynomials from $P_n(G)$ least deviating from zero in the uniform norm on a compact set K. We introduce and discuss the notion of Chebyshev constant $\tau(K, G)$ of compact set K with respect to G. We then consider the case when $G = \{z : |z| < R\}$ is the open disk. We study polynomials least deviating from zero in $L^p[-1, 1]$-means on $P_n(G)$. We also study a special case of Markov-Nikol'skii inequality on $P_n(G)$: the inequality between the L^p-mean ($p > 0$) of the kth derivative of a polynomial and the L^0-mean of this polynomial.

On the parabolic Cauchy problem for quantum graphs with vertex noise

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We consider the parabolic Cauchy problem associated with quantum graphs including Lipschitz or polynomial type nonlinearities and additive Gaussian noise perturbed vertex conditions. The vertex conditions are the standard continuity and Kirchhoff assumptions in each vertex. Our main result is that, in the case when only Kirchhoff conditions are perturbed, there exists a unique mild solution with continuous paths in the fractional domain space associated with the Hamiltonian operator of order less than $\frac{1}{4}$, and the solution is also Markov and Feller. These regularity properties are the quantum graph analogues of the case of a single interval and classical boundary Dirichlet or Neumann noise.

The talk is about a joint work with Mihály Kovács.

Viability and Filippov-type lemma for Stieltjes differential inclusions

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We prove a viability result for differential inclusions involving the Stieltjes derivative with respect to a left-continuous non-decreasing function $g : [0, 1] \to \mathbb{R}$

$$\begin{cases} x'_g(t) \in F(t, x(t)), \quad \mu_a - \text{a.e. } t \in [0, 1), \\ x(t) \in K(t), \quad \forall t \in [0, 1], \\ x(0) = x_0 \in K(0) \end{cases}$$

with time dependent state constraints. A tangential condition using the notion of contingent g-derivative is imposed. Classical viability results (for usual differential inclusions) are thus generalized and, at the same time, the gate to new viability results for difference inclusions, impulsive differential inclusions or dynamic inclusions on time scales is open. As a consequence, in the particular case where $K(t)$ is a tube, a Filippov-type lemma is obtained for this very general setting, of differential problems driven by Stieltjes derivatives.

Joint work with Bianca Satco.

Strong and weak associated reflexivity of certain function classes

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The report provides an overview of recent results on the problem of describing associated and doubly associated spaces to functional classes that include both ideal and non-ideal structures. The latter include first-order two-weight Sobolev spaces on the positive semiaxis [PSU19]. It is shown that, unlike the concept of duality, associativity can be "strong" and "weak". At the same time, the doubly associated spaces are divided into three more types. In this context, it is established that the space of Sobolev functions with a compact carrier has weakly associated reflexivity, and strongly associated with a weakly associated space consists only of zero [SU23]. Weighted spaces of Cesaro and Copson type have similar properties, for which the problem has been fully studied and their connection with Sobolev spaces with power weights [Ste22a] has been established. As an application, the problem of the boundedness of the Hilbert transformation from the Sobolev space to the Lebesgue space [Ste22b] is considered.

The study was performed at the Steklov Mathematical Institute and supported by the Russian Science Foundation (project 24-11-00170, https://rscf.ru/project/24-11-00170/).

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Cyclic-majorization and cyclically convexity

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In the presentation we are going to recall the concept of G-majorization between vectors. For the symmetric (permutation) group G, this definition leads to the one of the well-known standard ordering, which was introduced by Hardy, Littlewood and Pólya. In the main part of the talk we focus on another particular case, when G is the group of cyclic permutation. Applying the so-obtained cyclic majorization, one can define (cyclic) ordering preserving functions. Concerning these functions, that can be considered as certain Schur-convex functions, we present some characterization results.

Joint work with Judit Makó and Pál Burai.

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Representative product systems and harmonic analysis

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A more general and modern approach to the classical theory of Fourier series consists in the study of orthonormal systems defined on topological groups. A good example is the representation of the classical trigonometric system by its complex version on the torus. Another great example is the representation of the Walsh functions on the dyadic group, which is the topological group formed by bit sequences with logical addition, topology generated by dyadic intervals, and normalized Haar measure. A similar representation can also be made for Vilenkin groups and their systems. The dyadic group is formed by the complete product of cyclic group of order 2, but considering the complete product of arbitrary cyclic groups we obtain the Vilenkin groups. In this case we obtain an orthonormal and complete system (called Vilenkin system) if we take the characters of this structure ordered in the Paley's sense. Following the logic of the previous constructions, it is quite reasonable to ask ourselves how different it would be to consider the topological group formed by the complete product of finite groups that are not necessarily commutative. However, despite the similarity with Vilenkin's structures, the characters of a finite non-abelian group are orthonormal, but they do not form a complete system. This leads us to use representation theory to find the other members and thus obtain complete systems on finite groups. Product systems based on them are called representative product systems. In this talk we summarize the most relevant results with respect to the convergence of Fourier series based on representative product systems.

The Riemann–Liouville operators in weighted Besov spaces

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Properties of the Riemann–Liouville integration operators $I_{\pm}^{\alpha}$ of positive orders α [SKM93] in weighted Besov spaces on $\mathbb{R}$ are considered. Conditions are found for the fulfilment of inequalities connecting the norms of images and pre–images of $I_{\pm}^{\alpha}$. Spline wavelet systems and the corresponding decomposition theorems are used as tools for solving the problem. The results obtained are applied to the study of sequences of approximation and entropy numbers of $I_{\pm}^{\alpha}$, as well as to the study of the boundedness properties of the Hilbert transform.

The talk is based on publications [Ush21, Ush22, Ush23, Ush24]. The study was supported by the Russian Science Foundation (project 24-11-00170, https://rscf.ru/project/24-11-00170/).

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Some developments on the Wiener-Ikehara theorem

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The Wiener-Ikehara theorem is one of the cornerstones of complex Tauberian theory for Laplace transforms. This useful result has found many applications in diverse areas of mathematics such as number theory and spectral theory. In this talk we will survey some developments on the Wiener-Ikehara theorem from the last decade. Among others, we will discuss minimal assumptions on the boundary behavior of the Laplace transform, exact forms of the theorem, absence of reminders, and some quantified versions of it.

Quantum Wasserstein distances: metric properties and isometries

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Although the theory of classical optimal transport has been playing an important role in mathematical physics (especially in fluid dynamics) and probability since the late 80s, concepts of optimal transportation in quantum mechanics have emerged only very recently. First, we briefly review two such concepts: one relying on quantum channels (pioneered by De Palma and Trevisan) and one relying on quantum couplings (pioneered by Caglioti, Golse, Mouhot, and Paul). Then, we report on our progress in proving a conjecture of De Palma and Trevisan, saying that a smart modification of channel-based quantum Wasserstein distances gives rise to genuine metrics on quantum state spaces [BPTV24]. Finally, we describe the isometries of the qubit state space endowed with distinguished quantum Wasserstein distances [GPTV23].

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Quantitative 2D propagation of smallness and spectral estimates for Schrödinger operators

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We study the relation between propagation of smallness in the plane and control for heat equations. The former has been proved by Yuzhe Zhu who showed how the value of solutions in some small set propagates to a larger domain. By reviewing his proof, we establish a quantitative version with the explicit dependence of parameters. Using this explicit version, we establish new exact null-controllability results of 1D heat equations with any nonnegative power growth potentials $V \in \mathcal{C}(\mathbb{R})$. As a key ingredient, new spectral inequalities are established. The control set Ω that we consider satisfy

$$\left|\Omega \cap \left[x - L\langle x\rangle^{-s}, x + L\langle x\rangle^{-s}\right]\right| \geq \gamma^{\langle x\rangle^{\tau}} 2L\langle x\rangle^{-s}$$

for some $\gamma \in (0,1)$, $L > 0$, $\tau, s \geq 0$, and $\langle x\rangle := (1+|x|^2)^{1/2}$. In particular, the null-controllability result for the case of thick sets that allow the decay of the density (i.e., $s = 0$ and $\tau \geq 0$) is included. These extend the previous results from Ω being the union of equidistributive open sets to thick sets in the 1-dimensional case, and from bounded potentials to certain unbounded ones.

This work is part of my PhD under the supervision of Philippe Jaming.

On zeros of lacunary polynomials and completeness of certain exponential families

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Completeness and frame properties of the exponential systems in different function spaces are a classical subject of investigation. In particular, the systems of the form $\{t^k e^{2\pi i n t} : k = 0, \dots, N,\ n \in \mathbb{Z}\}$ are also well studied. We consider somewhat more general systems $E(S) := \{t^k e^{2\pi i n t} : k \in S,\ n \in \mathbb{Z}\}$ where S is a fixed subset of non-negative integers.

It turns out that both completeness and frame properties of $E(S)$ depend on the structure of the set S, and may substantially differ from the case $S = \{0, \dots, N\}$. This phenomenon is closely connected to certain properties of the zero sets of lacunary polynomials and generalized Vandermonde matrices.

The talk is based on joint work with A. Kulikov and A. Ulanovskii.

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Balayage of measures of finite energy on locally compact spaces

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This talk is based on [Zor22a]–[Zor23a], and it is devoted to the theory of potentials on a locally compact (Hausdorff) space X with respect to a *kernel* κ, κ being thought of as a symmetric, lower semicontinuous function $\kappa : X \times X \to [0, \infty]$. To be exact, we are interested in generalizations of the classical theory of balayage (sweeping out) on $\mathbb{R}^n$, $n \geqslant 2$ (see e.g. [Car46, Doo84]), to a suitable kernel κ on X.

We denote by $\mathfrak{M}$ the linear space of all (real-valued Radon) measures μ on X, equipped with the *vague* topology of pointwise convergence on the continuous functions $\varphi : X \to \mathbb{R}$ of compact support, and by $\mathfrak{M}^+$ the cone of all positive $\mu \in \mathfrak{M}$. (For the theory of measures and integration on X, we refer to Bourbaki [Bou04].) Given $\mu, \nu \in \mathfrak{M}$, the *mutual energy* and the *potential* are introduced by

$$I(\mu, \nu) := \int \kappa(x, y)\, d(\mu \otimes \nu)(x, y) \quad \text{and} \quad U^\mu(x) := \int \kappa(x, y)\, d\mu(y), \quad x \in X,$$

respectively, provided the value on the right is well defined as a finite number or $\pm\infty$. For $\mu = \nu$, the mutual energy $I(\mu, \nu)$ defines the *energy* $I(\mu, \mu) =: I(\mu)$ of $\mu \in \mathfrak{M}$.

In what follows, a kernel κ is assumed to satisfy the *energy principle*, which means that $I(\mu) \geqslant 0$ for all (signed) $\mu \in \mathfrak{M}$, and moreover that $I(\mu) = 0$ only for $\mu = 0$. Then all $\mu \in \mathfrak{M}$ of finite energy form a pre-Hilbert space $\mathcal{E}$ with the inner product $\langle \mu, \nu \rangle := I(\mu, \nu)$ and the energy norm $\|\mu\| := \sqrt{I(\mu)}$, cf. [Fug60, Lemma 3.1.2]. The topology on $\mathcal{E}$ introduced by means of this norm is said to be *strong*.

In addition, we shall always assume that κ satisfies the *consistency* principle, which means that the cone $\mathcal{E}^+ := \mathcal{E} \cap \mathfrak{M}^+$ is *complete* in the induced strong topology, and that the strong topology on $\mathcal{E}^+$ is *finer* than the vague topology on $\mathcal{E}^+$; such a kernel is said to be *perfect* (Fuglede [Fug60]). Thus any strong Cauchy net $(\mu_j) \subset \mathcal{E}^+$ converges *both strongly and vaguely* to the same unique measure $\mu_0 \in \mathcal{E}^+$.

Yet another permanent requirement on κ is that it satisfies the *domination principle*, which means that for any $\mu \in \mathcal{E}^+$ and any $\nu \in \mathfrak{M}^+$ with $U^\mu \leqslant U^\nu$ μ-a.e., the same inequality holds on all of X.

For any $A \subset X$, we denote by $\mathfrak{C}_A$ the upward directed set of all compact subsets K of A, where $K_1 \leqslant K_2$ if and only if $K_1 \subset K_2$. If a net $(x_K)_{K \in \mathfrak{C}_A} \subset Y$ converges to $x_0 \in Y$, Y being a topological space, then we shall indicate this fact by writing: $x_K \to x_0$ in Y as $K \uparrow A$.

Given $A \subset X$, we denote by $\mathfrak{M}^+_A$ the class of all $\mu \in \mathfrak{M}^+$ *concentrated on* A, which means that $X \setminus A$ is locally μ-negligible, or equivalently that A is μ-measurable and $\mu = \mu|_A$, $\mu|_A$ being the trace of μ to A. Also write $\mathcal{E}^+_A := \mathfrak{M}^+_A \cap \mathcal{E}$, and let $\mathcal{E}'_A$ stand for the closure of $\mathcal{E}^+_A$ in the strong topology on $\mathcal{E}$. Being a strongly closed subcone of the strongly complete cone $\mathcal{E}^+$, $\mathcal{E}'_A$ is likewise strongly complete.

Denoting by $c_*(E)$ and $c^*(E)$ the *inner* and *outer* capacity of $E \subset X$, respectively [Fug60, Section 2.3], we say that an assertion $\mathcal{A}(x)$ involving a variable point $x \in X$, holds *nearly everywhere* (*n.e.*), resp. *quasi-everywhere* (*q.e.*), on a set A if $c_*(E) = 0$, resp. $c^*(E) = 0$, where $E := \{x \in A : \ \mathcal{A}(x) \text{ fails}\}$.

Theorem 1. *For any $A \subset X$ and any $\sigma \in \mathcal{E}^+$, there exists $\sigma^A \in \mathcal{E}'_A$, called the inner balayage of σ to A, that is uniquely characterized by any one of the following (equivalent) assertions.*

(i) *σ^A is the unique measure in $\mathcal{E}'_A$ with the property $U^{\sigma^A} = U^\sigma$ n.e. on A.*

(ii) *σ^A is the unique orthogonal projection of σ in the pre-Hilbert space $\mathcal{E}$ onto the (convex, strongly complete) cone $\mathcal{E}'_A$. That is, $\sigma^A \in \mathcal{E}'_A$ and*

$$\|\sigma - \sigma^A\| = \min_{\mu \in \mathcal{E}'_A} \|\sigma - \mu\|.$$

(iii) *σ^A is the unique measure in $\mathcal{E}^+$ satisfying any one of the following three limit relations:*

$$\begin{aligned} \sigma^K &\to \sigma^A \quad \text{strongly in } \mathcal{E}^+ \text{ as } K \uparrow A, \\ \sigma^K &\to \sigma^A \quad \text{vaguely in } \mathcal{E}^+ \text{ as } K \uparrow A, \\ U^{\sigma^K} &\uparrow U^{\sigma^A} \quad \text{pointwise on } X \text{ as } K \uparrow A, \end{aligned}$$

where σ^K denotes the only measure in $\mathcal{E}^+_K$ with the property $U^{\sigma^K} = U^\sigma$ n.e. on K.

(iv) *σ^A is the only measure in the class $\Gamma_{A,\sigma}$ having the property*

$$U^{\sigma^A} = \min_{\mu \in \Gamma_{A,\sigma}} U^\mu \quad \text{on all of } X,$$

where $\Gamma_{A,\sigma} := \{\mu \in \mathcal{E}^+ : \ U^\mu \geqslant U^\sigma \quad \text{n.e. on } A\}$.

(v) *σ^A is the only measure in the class $\Gamma_{A,\sigma}$, introduced above, such that*

$$\|\sigma^A\| = \min_{\mu \in \Gamma_{A,\sigma}} \|\mu\|.$$

Corollary 2. *If X is representable as a countable union of compact sets, while κ satisfies Frostman's maximum principle, then σ^A is of minimum total mass in the class $\Gamma_{A,\sigma}$. That is,*

$$\sigma^A(X) = \min_{\mu \in \Gamma_{A,\sigma}} \mu(X). \tag{2}$$

Indeed, this follows at once from Theorem 1(iv) by applying Deny's principle of positivity of mass in the form stated in [Zor23b, Theorem 2.1].

Remark 3. *However, the extremal property (2) cannot serve as an alternative characterization of the inner balayage, for it does not determine σ^A uniquely. Indeed, consider the α-Riesz kernel $|x-y|^{\alpha-n}$ of order $\alpha \leqslant 2$, $\alpha < n$, on $\mathbb{R}^n$, $n \geqslant 2$, and a proper, closed subset A of $\mathbb{R}^n$ that is not α-thin at infinity [KM79, Definition 3.1] (take, for instance, $A := \{|x| \geqslant 1\}$). Then for any $\sigma \in \mathcal{E}^+_{A^c}$,*

$$\sigma^A \neq \sigma \quad \text{and} \quad \sigma^A(\mathbb{R}^n) = \sigma(\mathbb{R}^n), \tag{3}$$

the former being obvious e.g. from Theorem 1(ii), whereas the latter holds true by [Zor22b, Corollary 5.3]. Since $\sigma, \sigma^A \in \Gamma_{A,\sigma}$ while $\Gamma_{A,\sigma}$ is convex, combining (2) and (3) implies that there are actually infinitely many measures of minimum total mass in $\Gamma_{A,\sigma}$, for so is each $a\sigma + b\sigma^A$, where $a, b \in [0,1]$ and $a + b = 1$.

Theorem 4. *If a space X is second-countable, while a set A is Borel, then Theorem 1 remains valid with "n.e. on A" replaced throughout by "q.e. on A". The measure ω^{*A}, thereby uniquely determined, is said to be the outer balayage of ω to A. Actually, $\omega^{*A} = \omega^A$.* (Compare with [Fug16, Theorem 4.12].)

Remark 5. *All the above-mentioned assumptions on a space X and a kernel κ are fulfilled by:*

- ✓ *The α-Riesz kernels $|x-y|^{\alpha-n}$ of order $\alpha \in (0,2]$, $\alpha < n$, on $\mathbb{R}^n$, $n \geqslant 2$ (see [Lan72, Theorems 1.15, 1.18, 1.27, 1.29]).*
- ✓ *The associated α-Green kernels, where $\alpha \in (0,2]$ and $\alpha < n$, on an arbitrary open subset of $\mathbb{R}^n$, $n \geqslant 2$ (see [FZ18, Theorems 4.6, 4.9, 4.11]).*
- ✓ *The (2-)Green kernel on a planar Greenian set (see [Doo84, Sections I.V.10, I.XIII.7] and [Edw58]).*

(We emphasize that some of the results formulated above are new even for these classical kernels.)

Problem 6. *What kind of additional assumptions on X and κ would make it possible to generalize the above theory to Radon measures on X of infinite energy? (For a partial answer, see [Zor24b, Zor24a].)*

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On split representations of tensor products of von Neumann algebras

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A von Neumann algebra on a complex Hilbert space H is a $*$-subalgebra of the algebra $B(H)$ of all bounded linear operators on H, which contains the identity operator 1_H on H and is closed with respect to the weak operator topology.

If $M_1 \subset B(H_1)$, $M_2 \subset B(H_2)$ are two von Neumann algebras, then the algebraic tensor product $M_1 \otimes M_2$ can be identified with a $*$-subalgebra of $B(H_1 \overline{\otimes} H_2)$ by identifying $a_1 \otimes a_2 \in M_1 \otimes M_2$ with the operator in $B(H_1 \overline{\otimes} H_2)$ which maps the vector $\xi_1 \otimes \xi_2 \in H_1 \overline{\otimes} H_2$ in $(a_1\xi_1) \otimes (a_2\xi_2) \in H_1 \overline{\otimes} H_2$. The (von Neumann algebra) tensor product $M_1 \overline{\otimes} M_2$ of M_1 and M_2 is the von Neumann algebra generated by $M_1 \otimes M_2 \subset B(H_1 \overline{\otimes} H_2)$, that is its weak (or, equivalently, strong) operator closure.

If $N_j \subset B(K_j)$, $j = 1, 2$, are other von Neumann algebras and $\pi_j : M_j \longrightarrow N_j$, $j = 1, 2$, are $*$-isomorphisms, then the algebraic tensor product $*$-isomorphism $\pi_1 \otimes \pi_2 : M_1 \otimes M_2 \longrightarrow N_1 \otimes N_2$ has a (necessarily unique) extension to a $*$-isomorphism $\pi_1 \overline{\otimes} \pi_2 : M_1 \overline{\otimes} M_2 \longrightarrow N_1 \overline{\otimes} N_2$.

A spatial representation π of $M_1 \overline{\otimes} M_2$, that is a $*$-isomorphism of $M_1 \overline{\otimes} M_2$ onto a von Neumann algebra $\pi(M_1 \overline{\otimes} M_2) \subset B(H)$, is called split representation if it is unitarily equivalent to a spatial representation $\pi_1 \overline{\otimes} \pi_2$ as before : this means that there is a unitary $U : H \longrightarrow K_1 \overline{\otimes} K_2$ such that $\pi(a_1 \otimes a_2) = U^*\big(\pi_1(a_1) \otimes \pi_2)(a_2)\big)U$ for all $a_j \in M_j$, $j = 1, 2$. Intuitively : the representation space H "splits" into the tensor product of two Hilbert spaces, each one "supporting" one of the tensor factors M_1 and M_2.

We characterize all pairs (M_1, M_2) of von Neumann algebras having the property that any spatial representation of $M_1 \overline{\otimes} M_2$ is splitting.

The talk is based on joint work with Francesco Fidaleo.