

van Benthem characterization theorem for ultrafilter extensions

Zalán Molnár

Eötvös Loránd University

According to the well-known van Benthem characterisation theorem, a first order formula $\alpha(x)$ with one free variable x is equivalent to the standard translation of a modal formula if and only if $\alpha(x)$ is invariant for bisimulation [1]. This is often paraphrased as *modal logic is the bisimulation invariant fragment of first-order logic*. Since the original proof for basic modal languages, most modal extensions come up with their own characterizations. From the perspective of (basic) modal logics, there are three classes of structures that are of essential importance and are hard to overestimate. One is the class of finite models due to finite model property of (basic) modal logics, the second is the class of descriptive frames, which are the heart of general completeness and duality, and lastly the class of ultrafilter extensions that are analogues of saturated models and ultrapowers. On the one hand, [3] provides the characterization restricted to finite models, on the other, the recent paper of [2] proves it modulo descriptive frames. Motivated by [2] we show the characterization for the class of ultrafilter extensions.

- [1] J. van Benthem (1976). Modal correspondence theory. PhD thesis, University of Amsterdam.
- [2] N. Bezhanishvili, T. Henke (2020). A model-theoretic approach to descriptive general frames: the van Benthem characterization theorem. *Journal of Logic and Computation*, 30(7), 1331-1355.
- [3] E. Rosen. (1997). Modal logic over finite structures. *Journal of Logic, Language and Information*, 6(4), 427-439.