DENSIFICATION IN CLASSES OF INVOLUTIVE COMMUTATIVE RESIDUATED LATTICES

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The representation theorem for odd or even involutive FL_e-chains by bunches of layer groups, as discussed in [6], shall be redefined to demonstrate a more straightforward constructional relationship between odd or even involutive FL_e-chains and bunches of layer groups, bypassing the intermediary stage of layer algebras. By leveraging this redefined theorem, it is demonstrated that both the variety of semilinear odd involutive FL_e-algebras and its idempotent symmetric subvariety admits densification. Ultimately, employing the algebraic techniques introduced in [7], the proof of the strong standard completeness of Involutive uninorm logic with fixed point ($\mathbf{IUL}^{fp}$) is established, thus strengthening the main result of [5].

Varieties of FL-algebras (also known as pointed residuated lattices) serve as algebraic counterparts of substructural logics [1]. This area significantly intersects with the class of mathematical fuzzy logics [10]. Standard algebras for mathematical fuzzy logics are the ones from the corresponding variety which universe is the real unit interval $[0, 1]$. A mathematical fuzzy logic L (or the variety which is its equivalent algebraic counterpart) enjoys strong standard completeness if the following conditions are equivalent for each formula φ and theory T in L: (1) $T \vDash_L \varphi$, (2) for each standard L-algebra $\mathbf{A}$ and each $\mathbf{A}$-model e of T, e is an $\mathbf{A}$-model of φ. There are two weaker alternatives for defining standard completeness. The same definition but confining to finite theories yields the definition of finite strong standard completeness, whereas by setting $T = \emptyset$ one obtains the definition of (weak) standard completeness. The idea and the technique of proving strong standard completeness of a mathematical fuzzy logic via densification of countable chains followed by a subsequent Dedekind-MacNeille completion has been introduced in [7]. Since then this method has become folklore and has been extensively applied for proving strong standard completeness of a whole lot of substructural fuzzy logics in the literature. To exhibit one more application, we prove the strong standard completeness of $\mathbf{IUL}^{fp}$.

An original decomposition method along with the related (re)construction has been introduced in [6] for the class of odd or even involutive FL_e-chains. Its algebraic concept has been recently implemented to structurally describe various classes of residuated lattices, extending beyond its initial application. These encompass finite commutative, idempotent, and involutive residuated lattices, [8], finite involutive po-semilattices [9], and locally integral involutive po-monoids and semirings [2]. This representation has been lifted to a categorical equivalence in [3], and has proven to be a powerful weaponry to prove amalgamation results in classes of involutive FL_e-algebras which are neither integral, nor divisible, nor idempotent [4]. Using this representation, we prove the densification property for two varieties of odd involutive FL_e-algebras, namely for the semilinear one and its idempotent symmetric subvariety. Ultimately, we prove the strong standard completeness of Involutive Uninorm Logic with Fixed Point ($\mathbf{IUL}^{fp}$) in the algebraic style of [7], thereby surpassing the main result of [5].

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