LONG GAMES AND OPEN HYPERGRAPHS ON GENERALIZED BAIRE SPACES

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Descriptive set theory studies the properties of definable subsets of Polish spaces (e.g. the real line, the Baire space $^{\omega}\omega$ of functions $f : \omega \to \omega$ or the Cantor space $^{\omega}2$). Generalized descriptive set theory deals with analogues of these spaces associated to uncountable cardinals κ, and in particular with the generalized Baire space $^{\kappa}\kappa$ of functions $f : \kappa \to \kappa$. Many regularity properties for subsets of $^{\kappa}\kappa$, such as the perfect set property and the open graph dichotomy, are equivalent to the determinacy of two-player games of length κ.

In [1], Kechris introduced an abstract class of two-player games of countable length that encompasses many of the classical games characterizing regularity properties for subsets of Polish spaces. This talk focuses on the versions of Kechris's games of uncountable length for subsets of generalized Baire spaces. We show that the determinacy of these games follows from a recent generalization [2,3] of the open graph dichotomy to infinite dimensional directed hypergraphs. This is joint work in progress with Philipp Schlicht.

REFERENCES

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