Subalgebra Independence: The case of Subgroup Independence

Abstract

Motivated from a category-theoretic definition in a recent paper, "Subalgebra Independence", two subgroups of a group are **independent** if and only if, any two endomorphisms, one acting on each subgroup, can be extended to an endomorphism of the group generated by these subgroups. This definition illuminates that the usual condition of almost disjointness of subgroups (two subgroups A and B are almost disjoint if and only if $A \cap B = \{e\}$, where e is the identity element) is not enough to force independence and here we find necessary and (different) sufficient conditions for subgroup independence. We motivate the question of the full characterisation of subgroup independence as an interesting open problem. Moreover, we use the progress made so far to give a heuristic algorithm that decides subgroup independence for many cases.