

The translation invariant product measure problem in non sigma-finite case

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Abstract

Given two measure spaces, one may construct a product measurable space using the product sigma algebra. Suppose one of the two spaces is not sigma finite. Then, by the definition of product measure given in the introduction of Chapter 25 on Product Measures by D.H. Fremlin in the Measure Theory book Volume 2, a product measure may not necessarily be unique. For simplicity, assume that one of the measure spaces is the usual real measure space equipped with the Lebesgue measure, so that we may define translation through addition. The Lebesgue measure is known to be translation invariant. Translation by a real constant for a set in the product space is defined coordinate-wise. In this study, we give an example for a non translation-invariant product measure obtained from two translation invariant measures, one of which is non sigma-finite. Our particular example answers a question in the Bachelor thesis of the author for whether the translation invariant property generally holds for product measures without sigma-finiteness. Moreover, it also answers another question on the number of pairwise different product measures, which arose during the work, that there can be infinitely many product measures if we abandon the sigma-finiteness assumption.