Set membership with a few classical and quantum probes

Jaikumar Radhakrishnan
International Centre for Theoretical Sciences, Tata Institute of Fundamental Research,
Shivakote, Bengaluru 560089, Karnataka, India

We consider the following data structure problem. Given an n-element subset S of a universe of size m, represent S in memory as a bit string $x(S)$ so that membership queries of the form "Is x in S?" can be answered with a small number t of bit probes into $x(S)$. Let $s(m, n, t)$ be the minimum number of bits of memory, the length of $x(S)$, needed for this task. We will review the lower and upper bounds that are known for $s(m, n, t)$. We will then focus on the case $t = 2$, and present some recent upper and lower bounds for $s(m, n, t)$ in the classical and quantum settings. The arguments we use will be graph-theoretic, based on (i) dense graphs with large girth, and (ii) a theorem of Nash-Williams on covering the edges of a graph with forests. (The recent results were obtained in joint work with Shyam Dhamapurkar and Shubham Pawar.)