Scaling limit of SOS-type surfaces on a slope: From contracting Markov chains to the GFF

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The $(2+1)$D Solid-On-Solid (SOS) model famously exhibits a roughening transition: on an $N \times N$ torus with the height at the origin rooted at 0, the variance of $h(x)$, the height at a point x is $O(1)$ when the inverse-temperature β is large, vs $O(\log |x|)$ when β is small. The rigidity at large β is believed to fail once the surface is on a slope (tilted boundary conditions), which ought to destabilize it and induce the log-correlated behavior of the small β regime. The only rigorous result on this is by Sheffield (2005): if the slope θ is irrational, then $\mathrm{Var}(h(x))$ diverges with $|x|$ (with no known quantitative bound).

We study this model at a large fixed β, on an $N \times N$ torus with a nonzero boundary condition slope θ, perturbed by a potential V of strength $\epsilon(\beta)$ per site (arbitrarily small). Our main result is (a) the measure on height gradients ∇h has a weak limit μ as $N \to \infty$; and (b) the scaling limit of a sample from μ converges to a full plane GFF. In particular, we recover the asymptotics of $\mathrm{Var}(h(x))$. To our knowledge, this is the first example of a random surface that approximates the 3D Ising model interface, or any perturbation of one, where the scaling limit is recovered at large finite β under tilted boundary conditions.

The proof looks at random monotone surfaces that approximate the SOS surface, and derives their GFF limit via combinatorial and probabilistic tools; we will elaborate on one of these, relying on contraction properties of appropriately constructed Markov chains.

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