To what extent an ultrafilter extension can be similar to an ultrapower?

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The various connections between first-order and modal logic have always been a central topic in the model theory of modal logics. Our main purpose is to study general (first-order) model theoretic properties of (binary) relational structures having their roots in modal logic. The concept of ultrafilter extension is a fundamental one and has clear analogies with ultrapowers, hence the motto: *"ultrafilter extension in model theory of modal logics plays the rôle similar to ultrapowers in the first-order model theory"*. In general, these two structures are quite different from the perspective of first-order logic. In this talk we briefly present three very much related restrictions on the degree of the relation and show that the above mentioned constructions result important common model theoretical properties. Among others, is proved that these restrictions on the relation admit an elementary embedding of the ultrafilter extension into a suitable ultrapower. Also, we touch upon a case when elementary substructures are lifted to their ultrafilter extensions. At the end we take a short detour into certain Σ^1_1-fragments of second-order logic in order to comment and compair these theories of the two constuctions.