CONCEPT ALGEBRAS OF SPACETIME GEOMETRIES

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We define geometries over ordered fields, and we investigate the connections between their concept algebras (cylindric algebras). Minkowski spacetime (MinkST), Galilean spacetime (GalST) and Euclidean geometry (EuclG) are all special cases, and so is the extension of Galilean spacetime with light signals so that the speed of light is constant in a reference frame. The latter will be called late classical spacetime (LClassST). Let $\mathsf{Ca}(\mathrm{G})$ denote the concept algebra of geometry G. Then $\mathsf{Ca}(\mathrm{MinkST})$, $\mathsf{Ca}(\mathrm{GalST})$ and $\mathsf{Ca}(\mathrm{EuclG})$ are all proper subalgebras of $\mathsf{Ca}(\mathrm{LClassST})$, moreover, if the field is Euclidean and is elementarily equivalent to an Archimedean field, then they are maximal proper subalgebras of $\mathsf{Ca}(\mathrm{LClassST})$. In the proofs, the automorphism groups of geometries play an important role. For any two geometries G and G$'$, $\mathsf{Ca}(\mathrm{G})$ is a subalgebra of $\mathsf{Ca}(\mathrm{G}')$ if and only if $\mathsf{Aut}(\mathrm{G})$ contains $\mathsf{Aut}(\mathrm{G}')$, where $\mathsf{Aut}(\mathrm{G})$ denotes the automorphism group of geometry G. We prove the results mentioned above by showing that $\mathsf{Aut}(\mathrm{LClassST})$ is a maximal proper subgroup of the automorphism groups of MinkST, GalST and EuclG in the case of Archimedean Euclidean fields. It is interesting that for any non-Archimedean field, $\mathsf{Aut}(\mathrm{LClassST})$ is not a maximal proper subgroup of the automorphism groups of the other three geometries, but there are non-Archimedean fields for which the concept algebras of these three geometries are maximal subalgebras of $\mathsf{Ca}(\mathrm{LClassST})$. We also investigate how these results depend on the ordered field and the dimension of spacetime, and we will mention some open problems.