Propositional versus predicate logics: an application of universal algebraic logic.

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In a propositional logic, atomic formulas stand for arbitrary formulas of the logic, this is expressed in the so-called substitutional property.: A logic is called substitutional if whenever a formula is valid, all of the formulas obtained from it by substituting arbitrary formulas in the place of atomic ones are valid, too. In other words, a formula is valid iff it is valid as a formula scheme. Propositional logics are the subject of investigation of Abstract Algebraic Logic, see [1].

As opposed to propositional logics, in a predicate logic the atomic formulas do not stand for arbitrary formulas. For example, in first-order logic (FOL), the atomic formula $R(x)$ stands for an arbitrary formula with one free variable x. This is expressed in the notion of conditional substitutional property ([2], Def.3.3.14). Not all logics are conditionally substitutional. We call logics that satisfy the conditional substitutional property predicate logics. The theory of predicate logics is richer than the theory of propositional logics. For example, in a predicate logic, the notions of formulas and formula schemes are separated.

Each predicate logic has a well-defined propositional core, a propositional logic that can be called the sentential level of the logic. Using theorems from [2], we will show the following. The propositional core of FOL is the so-called type-free logic (see, e.g., [3] and [4, sec.4.3]). This core is not compact. We also show that the compact propositional core of FOL is the full finitary logic of infinitary relations (see [4, sec.4.3]). To our minds, these kinds of investigations show that algebraization of FOL as outlined in [5] and pursued further in [4] and [2] is successful and rich in further possibilities.

References

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