Approaching Vaught's Conjecture using Algebraic Logic

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Abstract

In this talk, we indicate a way of distinguishing between (what we call) Henkin ultrafilters of locally finite cylindric and quasi-polyadic algebras, for which two ultrafilters are said to be distinguishable. We give a result about the number of so-called distinguishable (distinct in some sense) ultrafilters in a given locally finite countable algebra. In model theoretic terms, such ultrafilters represent intrinsically potential models of a theory T, when the algebra at hand is represented as Fm_T; the Tarski–Lindenbaum cylindric algebra of T. If two models are elementary equivalent they are not distinguishable (but the converse may fail), and 'not distinguishable' is strictly weaker than being isomorphic. Our first main thereby model-theoretic result obtained this way is that for any countable first order theory T in a countable language, with or without equality, if it has an uncountable set of countable models that are pairwise distinguishable, then such a set has exactly $2^{\aleph_0}$ pairwise distinguishable models.

We also count non-isomorphic models that omit a countable given family of non principal types of a theory, and we get the same cardinals provided by Morley's Theorem on Vaught's conjecture. We give an example of a theory T that has only one model omitting a given family of non-principal types (namely the prime model) and continuum many non-isomorphic models. We investigate an analogue of Vaught's conjecture for a natural proper extension of first order languages called rich languages, studied frequently in algebraic logic counting so-called weak models (arising naturally from the notion of weak cylindric set algebras). We show that the number of weak non-isomorphic (in the ordinary sense) models having a finite signatures formulated in a rich language satisfies Vaught's conjecture, even when we count the number of weak models omitting a (given in advance) countable family of non principal types in a countable theory. This approach seems to open a promising fruitful avenue between Algebraic Logic and deep Model Theory.