Tarskian Algebraic Logic, Relativity Theory and Methodology of Science

István Németi is 80

15 September - 17 September 2022 (online)

Volume of Abstracts

Alfréd Rényi Institute of Mathematics 2022

Tarskian Algebraic Logic, Relativity Theory and Methodology of Science

September 15-17, Budapest 2022 (online)

István Németi is turning to be 80 in 2022. We are pleased to announce that we are organizing a conference in the Alfréd Rényi Institute of Mathematics honoring this occasion. The event contains 3 consecutive afternoons. Each day is focusing on a theme of István's main research topics (Algebraic Logic, Relativity Theory, Methodology of Science). The conference will be closed by a roundtable discussion about the future of algebraic logic and its possible applications.

https://conferences.renyi.hu/nemeti80

This conference is organized by **Alfréd Rényi Institute of Mathematics** in Budapest, and will be hosted virtually through an online platform. The concrete link needed to join the online event will be sent using the e-mails given during the registration.

All talks will be given by invited speakers.

Organizers & Program Committee:

- Hajnal Andréka (Rényi Institute)
- Judit X. Madarász (Rényi Institute)
- Gergely Székely (Rényi Institute)

Invited Speakers (Talks):

- Ági Kurucz (King's College London)
- Zalán Gyenis (Jagiellonian University)
- Tarek Sayed Ahmed (Cairo University)
- Robin Hirsch (University College London)
- Peter Jipsen (Chapman University)
- Márton Gömöri (Eötvös Loránd University)
- Judit X Madarász (Rényi Institute)
- Joshua Babic & Lorenzo Cocco (Université de Genève)
- JB Manchak (University of California, Irvine)
- Hans Halvorson (Princeton University)
- Laurenz Hudetz (London School of Economics)
- Mohamed Khaled (Bahçeşehir University)

Invited Speakers (Roundtable discussion):

- Hajnal Andréka & István Németi (Rényi Institute)
- Michele Friend (Université de Lille, George Washington University)
- Roger D Maddux (Iowa State University)
- Vaughan Pratt (Stanford University)
- Ildikó Sain (Rényi Institute)
- Gábor Sági (Rényi Institute)

Schedule and Program

Day 1 (Algebraic Logic), 15 September 2022

15:00-15:30	**Ági Kurucz** Non-finitely axiomatisable canonical varieties of 'non-relativised' algebras of relations with infinite canonical axiomatisations	Chair: **Mohamed Khaled**
15:30-16:00	**Zalán Gyenis** Another look on amalgamation in algebraic logic	
16:00-16:30	**Tarek Sayed Ahmed** Approaching Vaught's Conjecture using Algebraic Logic	
16:30-17:00	Break	
17:00-17:30	**Robin Hirsch** Temporal Logic of Minkowski spacetime	Chair: **Zalán Gyenis**
17:30-18:00	**Peter Jipsen** Weakening Relation Algebras	

Day 2 (Relativity Theory) 16 September 2022

15:00-15:30	**Márton Gömöri** Bell's spaceships in free fall	Chair: **Gergely Székely**
15:30-16:00	**Judit X Madarász** Concept algebras of geometries with affine reducts over ordered fields	
16:00-16:30	Break	
16:30-17:00	**Joshua Babic** & **Lorenzo Cocco** Equivalent Formulations of Special Relativity	Chair: **Márton Gömöri**
17:00-17:30	**JB Manchak** A Hierarchy of Spacetime Symmetries: Holes to Heraclitus	

Day 3 (Methodology of Science) 17 September 2022

15:00-15:30	**Hans Halvorson** Translations between Translations	Chair: **Michele Friend**
15:30-16:00	**Laurenz Hudetz** A unified view of theories	
16:00-16:30	**Mohamed Khaled** A conceptual-based attribute to connections between theories	
16:30-17:00	Break	
17:00-18:30	Roundtable discussion: On the future of Tarskian algebraic logic and its possible applications Main participants: **Hajnal Andréka** & **István Németi** **Michele Friend** **Roger D Maddux** **Vaughan Pratt** **Ildikó Sain** **Gábor Sági**	Chair: **Gergely Székely**

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Invited Talks

Non-finitely axiomatisable canonical varieties of 'non-relativised' algebras of relations with infinite canonical axiomatisations

Ági Kurucz

Canonicity is a central notion in the theory of Boolean algebras with normal and additive operators (BAOs), and it is an important tool in proving Kripke completeness of propositional multimodal logics. Though in general canonicity of an equation is an undecidable 'semantical' property, there exist well-known syntactical classes of canonical equations, such as Sahlqvist equations and their generalisations by Goranko and Vakarelov.

While any set of canonical equations clearly axiomatises a canonical variety, the converse does not always hold for non-finitely based varieties. Well-known counterexamples are algebraisations of finite-variable fragments of classical first-order logic (representable relation algebras and cylindric algebras of dimension ≥ 3, with or without diagonals). These are canonical varieties that are only barely canonical in the sense that every base for their equational theories must contain infinitely many noncanonical equations. On the other hand, for dimension 2, the situation is simpler: say, the equational theory of the variety of two-dimensional representable diagonal-free cylindric algebras (the algebraic counterparts of two-variable substitution and equality free first-order logic) does have a finite Sahlqvist axiomatisation.

The question arises whether there are varieties "in between" the two extremes: canonical varieties that are non-finitely based but still possess an infinite canonical axiomatisation. A well-known example "in between" is the variety Crs_n of cylindric-relativised set algebras, for $n \geq 3$.

In this talk we answer the question affirmatively even for varieties of 'non-relativised' algebras of relations, by considering various algebraisations of two-variable substitution and equality free first-order logic extended with 'elsewhere' quantifiers.

Another look on amalgamation in algebraic logic

Zalán Gyenis

There is a well-established correspondence between interpolation and amalgamation for algebraizable logics that satisfy certain additional assumptions (such as conjunctiveness, compactness and having a deduction term). In this talk we introduce the Robinson property of a logic and show that a conditionally algebraizable logic without any of the additional assumptions has the Robinson property if and only if the corresponding class of L indenbaum-Tarski algebras has the amalgamation property. Moreover, we give the logical characterization of the strong amalgamation property, solving an open problem of Andréka–Németi–Sain. Given the mentioned extra assumptions the Robinson property implies the interpolation property. As conditionally algebraizable logics cover algebraizable logics as well as various quantifier logics such as classical first order logic, our results yield a generalization of some of the results concerning interpolation and amalgamation.

Approaching Vaught's Conjecture using Algebraic Logic

Tarek Sayed Ahmed

In this talk, we indicate a way of distinguishing between (what we call) Henkin ultrafilters of locally finite cylindric and quasi-polyadic algebras, for which two ultrafilters are said to be distinguishable. We give a result about the number of so-called distinguishable (distinct in some sense) ultrafilters in a given locally finite countable algebra. In model theoretic terms, such ultrafilters represent intrinsically potential models of a theory T, when the algebra at hand is represented as Fm_T; the Tarski-Lindenbaum cylindric algebra of T. If two models are elementary equivalent they are not distinguishable (but the converse may fail), and 'not distinguishable' is strictly weaker than being isomorphic. Our first main thereby model-theoretic result obtained this way is that for any countable first order theory T in a countable language, with or without equality, if it has an uncountable set of countable models that are pairwise distinguishable, then such a set has exactly $2^{\aleph_0}$ pairwise distinguishable models.

We also count non-isomorphic models that omit a countable given family of non principal types of a theory, and we get the same cardinals provided by Morley's Theorem on Vaught's conjecture. We give an example of a theory T that has only one model omitting a given family of non-principal types (namely the prime model) and continuum many nonisomorphic models. We investigate an analogue of Vaught's conjecture for a natural proper extension of first order languages called rich languages, studied frequently in algebraic logic counting so-called weak models (arising naturally from the notion of weak cylindric set algebras). We show that the number of weak non-isomorphic (in the ordinary sense) models having a finite signatures formulated in a rich language satisfies Vaught's conjecture, even when we count the number of weak models omitting a (given in advance) countable family of non principal types in a countable theory. This approach seems to open a promising fruitful avenue between Algebraic Logic and deep Model Theory.

Temporal Logic of Minkowski spacetime

Robin Hirsch

Dialogue

Me: how many spatial dimensions do we have?

You: three.

Me: how do you know its not less than three?

You: because I can make a regular tetrahedron where all sides have equal lengths, I couldn't do this with only two spatial dimensions.

Me: OK, I'm convinced. So can you write down a temporal formula which holds with three spatial dimensions but not with only two spatial dimensions?

You: let me think about that.

Abstract

According to relativity theory, the world is made up of space-time points which can send signals to each other at up to and including the speed of light. One striking difference with a Galilean model, is that there is no notion of simultaneity in relativity theory. So the Kripke frame where the worlds are space-time points and the accessibility is 'can send a signal to', branches densely in the future and in the past.
For special special relativity the Kripke frame is $\left(R^{n+1}, <\right)$ with one time dimension and n spatial dimensions, where two spacetime points are ordered by $<$ if and only if it is possible to send a signal from the first spacetime point to the second. For fixed n, there are four cases to distinguish according to whether this ordering is reflexive or irreflexive and whether signals may be sent at up to the speed of light, or strictly less than the speed of light. Temporal propositional formulas are built from propositions with propositional connectives and temporal operators F, P, G, H (sometime in the future, past, always in the future, past).
For each frame $\left(R^{n+1}, <\ \right)$ (where $<$ is reflexive/irreflexive, signals can/cannot go at speed of light, no. of spatial dimensions $n = 1, 2, 3, \ldots$) we consider three problems.

1. Distinguish these frames from each other by temporal formulas.
2. Find an axiomatisation of the temporal validities over each frame.
3. For each frame, is the validity problem for temporal formulas over the frame decidable, what is the complexity?

Very limited progress has been made with problems 1 and 2 . We do have a few results for problem 3 (when $n = 1$ the complexity is PSPACE complete, when $n > 1$ the complexity is EXPTIME hard, whether $<$ is reflexive or irreflexive, whether speed of light signals are allowed or not).

Weakening Relation Algebras

Peter Jipsen

In honor of Istvan Nemeti's 80th birthday, this talk will focus on algebras of binary relations that are slightly more general than representable relation algebras. Let $\leq$ be a partial order on a set X. A binary relation R on X is a weakening relation with respect to $\leq$ if $\leq; R \leq \subseteq R$. Weakening relations have applications in sequent calculi, proximity lattices/spaces, order-enriched categories,

intuitionistic modal logic, mathematical morphology and program semantics, e.g. via separation logic.

The set of all weakening relations on $(X, \leq)$ is denoted by $\mathrm{Wk}(X, \leq)$, and it is closed under the operations of composition (;), union ($+$), intersection ( ), complement-converse ($\sim R = \neg R^{\smile}$) and has $\leq$ as identity element for composition. The algebra $\mathrm{wk}(\leq) = (\mathrm{Wk}(X, \leq), +, \cdot, \emptyset, \top, ;, \sim, 1)$, where $\top = X^2$ and $1 = \leq$, is called a full representable weakening relation algebra, and the class of all subalgebras of products of such algebras is denoted by RwkRA. The set $\mathrm{Wk}(X, \leq)$ is also closed under a Heyting implication

$$R \to S = \{(x, y) \mid \forall u, v(u \leq x \& y \leq v \& uRv \Longrightarrow uSv)\}$$

and if this operation is added then the resulting algebra, denoted $\mathbf{Wk}(\leq)$, is a cyclic involutive generalized bunched implication algebra and the SP-closure of this class is denoted by RWkRA.

In this talk we will present what is currently known about RwkRA and RWkRA, and how they are related to the variety RRA of representable relation algebras. The results about RWkRA are based on joint research with Nick Galatos [1,2] and the more recent results about RwkRA are based on joint research with Jaš Šemrl.

REFERENCES

[1] N. Galatos and P. Jipsen, The structure of generalized BI-algebras and weakening relation algebras, Algebra Universalis, (2020) 81: 35, 1-35

[2] N. Galatos and P. Jipsen, Weakening relation algebras and FL^2-algebras, in proc. 18th RAMiCS conference, LNCS Vol 12062, Springer (2020), 117-133

Bell's spaceships in free fall

Márton Gömöri

Two small spaceships, initiated from the state of rest, fall freely in a uniform gravitational field, in a way that the direction of their separation is parallel with the field. As the space ships are accelerated by a uniform field, they will have at every moment the same velocity, and so remain displaced one from the other by a fixed distance. Suppose that a fragile thread is tied initially between the spaceships. If it is just long enough to span the required distance initially, then as the rockets speedup, it will be come too short, because of its need to Lorentz contract,and must finally break. It must break when, at a sufficiently high velocity, the prevention of the natural contraction imposes intolerable stress.

Is it really so?

Concept algebras of geometries with affine reducts over ordered fields

Judit X Madarász

We define geometries over ordered fields, and we investigate the concept algebras of these geometries. Affine geometry, Euclidean geometry, geometry of Minkowski spacetime and that of classical spacetime are all special cases.

Equivalent Formulations of Special Relativity

Joshua Babic & Lorenzo Cocco

We will compare a geometric system for Minkowski spacetime with the hungarian school's axiomatization of special relativity in terms of observers and coordinate systems. We will argue that they are theoretically equivalent: two presentations of the same theory. We will show in particular that they are Moritaequivalent, but not definitionally equivalent. This provides corroboration for the thesis that Morita equivalence is the right standard of equivalence. However, minor modifications to the hungarian dynamical approach will be recommended on philosophical grounds. The implications of these results will then be briefly discussed.

A Hierarchy of Spacetime Symmetries: Holes to Heraclitus

JB Manchak

We present the hierarchy of symmetry conditions within the context of general relativity. The weakest condition captures a sense in which space- time is free of symmetry "holes" of a certain type. All standard models of general relativity satisfy the condition but we show that violations can occur if the Hausdorff assumption is dropped. On the other extreme, the strongest condition of the hierarchy is satisfied whenever a model is com- pletely devoid of symmetries. In these "Heraclitus spacetimes" no pair of distinct points can be mapped (even locally) into one another. The condition is incredibly strong but we show that Heraclitus spacetimes do, in fact, exist. We close with a brief comment on the prospect of using the symmetries of a spacetime as a guide to how much "structure" it possesses.

Translations between Translations

Hans Halvorson

Mathematical evidence suggests that it is most natural to treat the collection of all theories (properly formalized) as forming a 2- category rather than just a 1-category. But this means that there are mathematical objects, viz. the 2-cells between translations, that philosophers of science have not been talking about. In this talk, I explain what 2-cells are (mathematically) and what significance they might have for foundational debates.

A unified view of theories

Laurenz Hudetz

Logic-based approaches to analysing physical theories have become somewhat unpopular among philosophers of science over the past decades. An important reason for this is the widespread view that hardly any actual physical theories (that are commonly formulated in terms of differential equations) can be represented in a logic-based way. I argue that this worry can be overcome and that much can be gained from logic-based analyses of theories. I outline a general logic-based approach to representing theories that are based on differential equations.

A conceptual-based attribute to connections between theories

Mohamed Khaled

We distill the idea of 'concepts' from the pioneering work of Alfred Tarski. We argue that, in Tarski's understanding, a concept is just a relation that can be defined on a given theory using its own language. These concepts define an algebra in a natural way; we refer to it as the concept algebra of the theory in hands. This perspective goes back to George Boole in his creation of Boolean algebras.

There is a vast literature deliberating that properties of a theory can be read off from its concept algebra. In this talk, we will introduce a new understating of the interrelationship between theories based on studying some natural algebraic relations between their concept algebras. We will show

how this new insight can be used to obtain better understanding of the connections between some particular theories of physics.
