ON THE FUTURE OF RA

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Hilbert, Erdös, Tarski, and Jónsson have demonstated how unsolved problems can influence the future of mathematics. For relation algebras, many problems from [4] (1994) have been solved, but some remain whose solutions would be extremely illuminating. Here are three conjectures, first named and then stated with comments.

(1) the Flexible Atom (or Trio) Conjecture,
(2) the Random Representability Conjecture,
(3) the Ramsey Algebra Conjecture.

(1) Every finite integral relation algebra with a **flexible atom** or a **flexible trio** (of atoms) is representable on a *finite* set (not just infinite sets).

Flexible atoms and trios (of atoms) provide simple strategies for extending any partial representation by one point, ensuring representation on an infinite set. Special cases of the Flexible Atom Conjecture (1985) have been proved by probabilistic methods. For the Flexible Trio Conjecture (2018) see [5, Def. 4.5, Prob. 4.6].

(2) A **random** finite relation algebra is *almost certainly* **representable**.

[3, Th.15] (1984): If V is a *finitely axiomatized subvariety* of RRA (the non-finitely-based variety of representable relation algebras) then a random finite relation algebra is *almost certainly* in V: if $\mathsf{RA}(n)$ is the number of isomorphism types of relation algebras with n atoms and $V(n)$ is the number of isomorphism types of algebras in V with n atoms then $\lim_{n\to\infty} \frac{V(n)}{\mathsf{RA}(n)} = 1$. Prove this when $V = \mathsf{RRA}$.

(3) Every **Ramsey algebra** is **representable**. [2, Prob. 2.7]

The Ramsey algebra $\mathfrak{E}_n^{\{2,3\}}$ is a relation algebra with n atoms that is symmetric ($\breve{x} = x$), integral (identity element 1' is an atom), $a;b = \overline{1'}$, and $a;a = \overline{a}$) for distinct diversity atoms a, b. The size of a representation of $\mathfrak{E}_n^{\{2,3\}}$ is bounded by the Ramsey Theorem that the edges of a large complete graph cannot be colored with a few colors without creating a monochromatic triangle.[1] Ramsey algebras are known to be representable if the number of atoms is less than 2000 and not equal to 8 or 13, where a certain method does not work. Trotter-Erdös-Szemerédi "proved" by probabilistic methods in the 1980s that every sufficiently large Ramsey algebra is representable.[2]

References

[1] R. C. Lyndon. Relation algebras and projective geometries. *Michigan Math. J.*, 8:21–28, 1961.

[2] Roger Maddux. Some varieties containing relation algebras. *Trans. Amer. Math. Soc.*, 272(2):501–526, 1982.

[3] Roger Maddux. Finite integral relation algebras. In *Universal algebra and lattice theory (Charleston, S.C., 1984)*, volume 1149 of *Lecture Notes in Math.*, pages 175–197. Springer, Berlin, 1985.

[4] Roger D. Maddux. A perspective on the theory of relation algebras. *Algebra Universalis*, 31(3):456–465, 1994.

[5] Roger D. Maddux. Subcompletions of representable relation algebras. *Algebra Universalis*, 79(2):Paper No. 20, 32, 2018.

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[1] Another hard combinatorial problem arises by changing "2" to "1": the relation algebra $\mathfrak{E}_{n+1}^{\{1,3\}}$ is representable iff there is a projective plane of order $n+2$ [1, Lyndon 1961].

[2] In their construction, everything that could occur eventually does occur but what needs to occur cannot ever occur.