Non-finitely axiomatisable canonical varieties of 'non-relativised' algebras of relations with infinite canonical axiomatisations

Agi Kurucz*

Abstract

Canonicity is a central notion in the theory of Boolean algebras with normal and additive operators (BAOs), and it is an important tool in proving Kripke completeness of propositional multimodal logics. Though in general canonicity of an equation is an undecidable 'semantical' property, there exist well-known syntactical classes of canonical equations, such as Sahlqvist equations and their generalisations by Goranko and Vakarelov.

While any set of canonical equations clearly axiomatises a canonical variety, the converse does not always hold for non-finitely based varieties. Well-known counterexamples are algebraisations of finite-variable fragments of classical first-order logic (representable relation algebras and cylindric algebras of dimension ≥ 3, with or without diagonals). These are canonical varieties that are only barely canonical in the sense that every base for their equational theories must contain infinitely many non-canonical equations. On the other hand, for dimension 2, the situation is simpler: say, the equational theory of the variety of two-dimensional representable diagonal-free cylindric algebras (the algebraic counterparts of two-variable substitution and equality free first-order logic) does have a finite Sahlqvist axiomatisation.

The question arises whether there are varieties "in between" the two extremes: canonical varieties that are non-finitely based but still possess an infinite canonical axiomatisation. A well-known example"in between" is the variety Crs_n of cylindric-relativised set algebras, for $n \geq 3$.

In this talk we answer the question affirmatively even for varieties of 'non-relativised' algebras of relations, by considering various algebraisations of two-variable substitution and equality free first-order logic extended with 'elsewhere' quantifiers.

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