Celebrating István Németi’s 80th Birthday

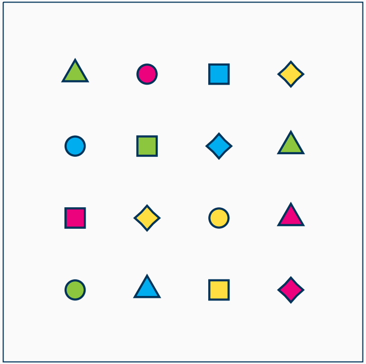
A conceptual-based attribute to connections between theories

Mohamed Khaled


CLASSROOM A

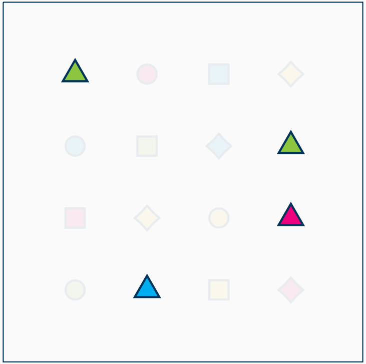
Concept algebras ...

Propositional Structure



M

Propositional Structure



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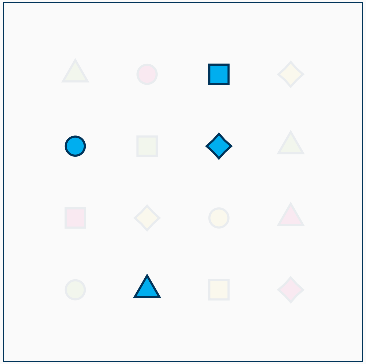

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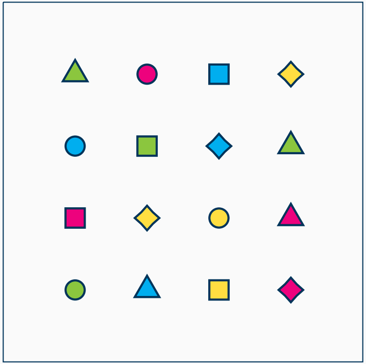

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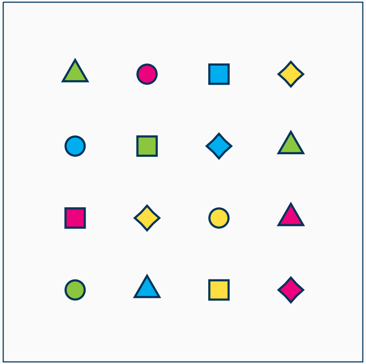

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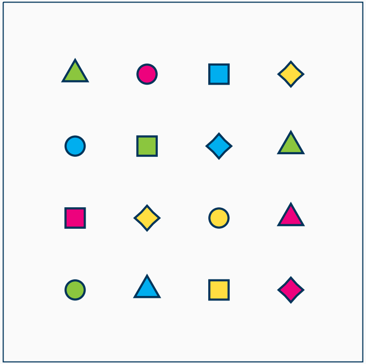

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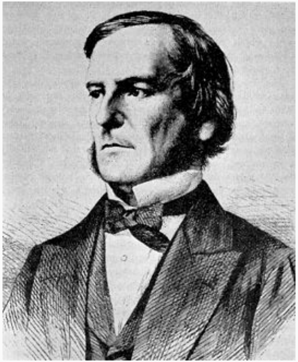
$\wedge$, $\vee$, $\neg$

Boolean Algebras



Goerge Boole (1815 - 1864)

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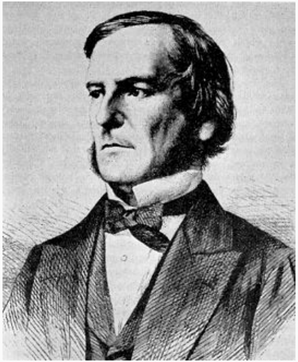

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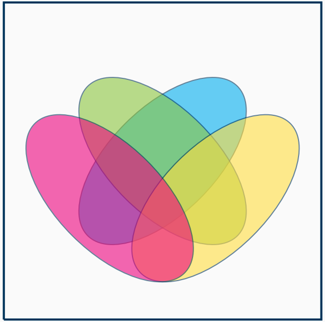

Boolean set algebras

Variety of BA’s

$$2 \stackrel{\text{def}}{=} \langle \{0,1\}, \wedge, \vee, \neg, 0, 1 \rangle$$

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

p	q	$p \vee q$
0	0	0
0	1	1
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p	$\neg p$
0	1
1	0

First-Order Structures

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$$\frac{}{M; x}$$

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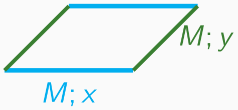

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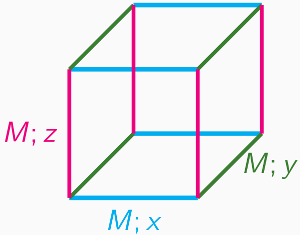

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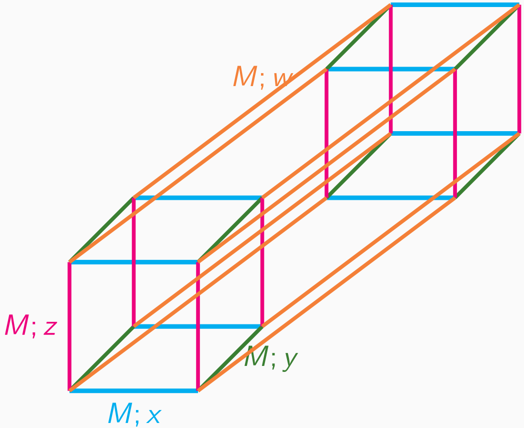

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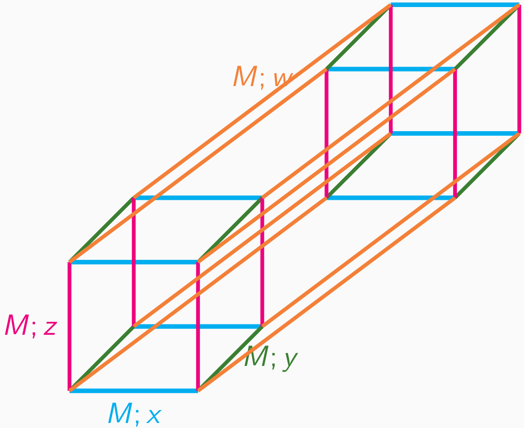

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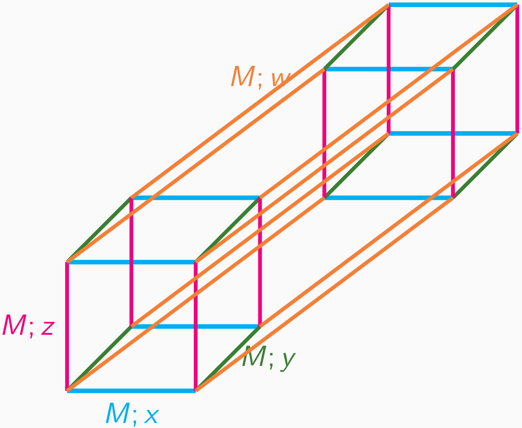

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$M, M^2, M^3, M^4, \ldots$

Space of Concepts

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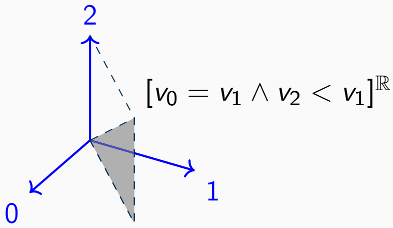

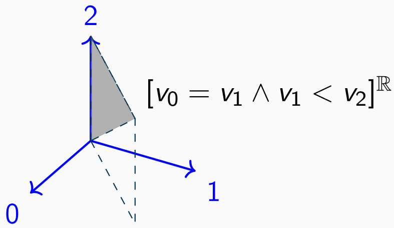

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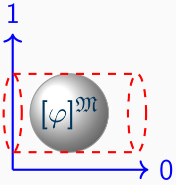

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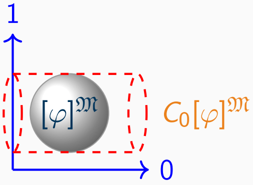

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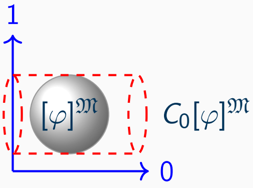

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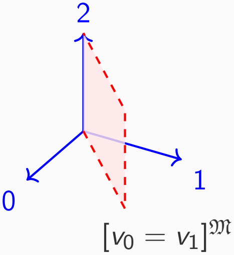

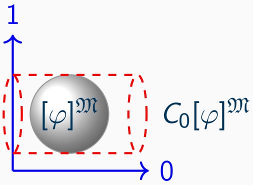

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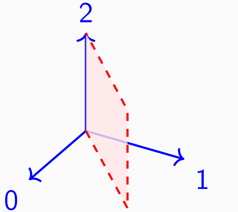
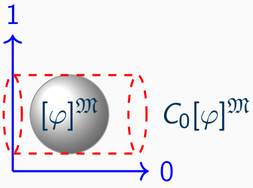


$D_{01} = [v_0 = v_1]^{\mathfrak{M}}$



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Definition

The concept algebra of $\mathfrak{M}$ is:

$$\mathfrak{Cs}(\mathfrak{M}) \stackrel{\text{def}}{=} \langle Cs(\mathfrak{M}), \cup, \sim, C_i, D_{ij} \rangle_{i,j<\omega}.$$

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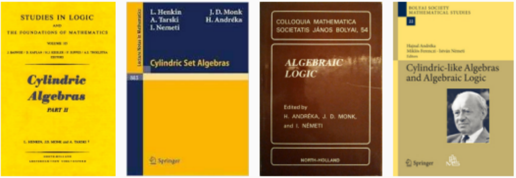
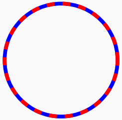
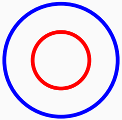


Connections between theories …

Subalgebras

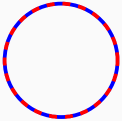


$\mathfrak{Cs}(\mathfrak{M}) = \mathfrak{Cs}(\mathfrak{N})$



$\mathfrak{Cs}(\mathfrak{N}) \subseteq \mathfrak{Cs}(\mathfrak{M})$

Subalgebras



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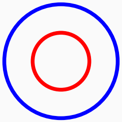
$$\Updownarrow$$

$$\mathfrak{M} \triangleq \mathfrak{N}$$



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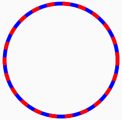

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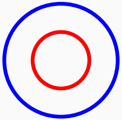

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Homomorphisms

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A translation from the language of $\mathfrak{M}$ to the language of $\mathfrak{N}$

Homomorphisms

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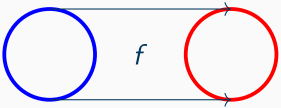

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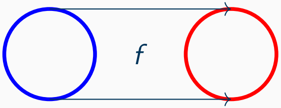

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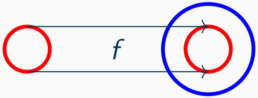

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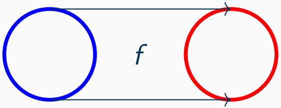

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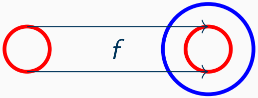

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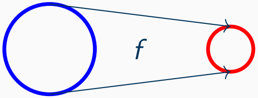

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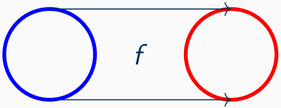
Homomorphisms



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$$\Updownarrow$$
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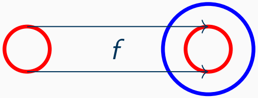

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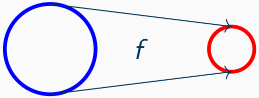

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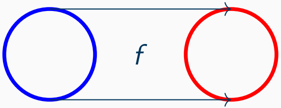
Homomorphisms



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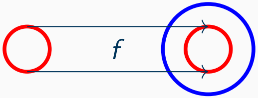

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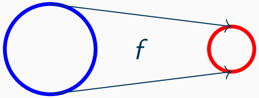

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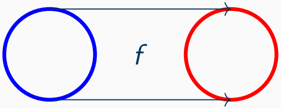

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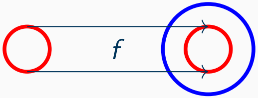

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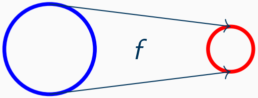

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Definitional Equivalence $\Longrightarrow$ Mutual Definability

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$\Longleftarrow$

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$\not\Longleftarrow$

H. Andréka, J. Madarász and I. Németi (2005)

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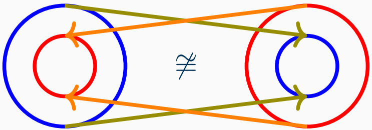
Definitional Equivalence $\Longrightarrow$ Mutual Definability

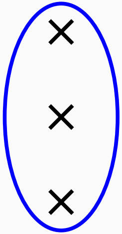
$\not\Longleftarrow$



H. Andréka, J. Madarász and I. Németi (2005)

Generating Sets

Generating Sets



$$\mathfrak{Cs}(\mathfrak{M}) = \langle X \rangle$$

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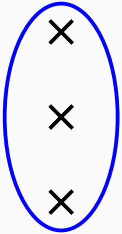

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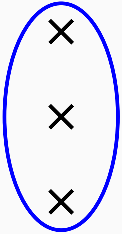
Changing the language of $\mathfrak{M}$

Generating Sets



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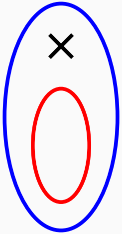

$$\mathfrak{Cs}(\mathfrak{M}) = \langle \mathfrak{Cs}(\mathfrak{N}), X \rangle$$

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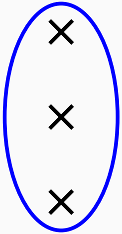

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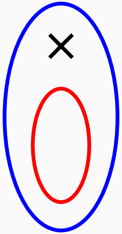

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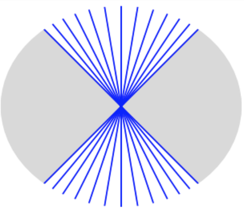
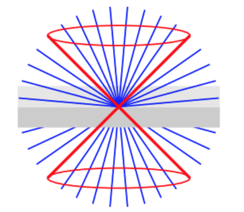
Adding concepts to $\mathfrak{N}$ to get $\mathfrak{M}$

Example:

$$\mathcal{RS} = \langle \mathbb{R}^4, \mathsf{col}^t \rangle$$



$$\mathcal{CS}^+ = \langle \mathbb{R}^4, \mathsf{col}^\infty, \mathsf{col}^\lambda \rangle$$



Conjecture (H. Andréka)

$$\mathfrak{Cs}(\mathcal{CS}^+) = \left\langle \mathfrak{Cs}(\mathcal{RS}), b \right\rangle$$

for any concept b in $\mathfrak{Cs}(\mathcal{CS}^+)$ not in $\mathfrak{Cs}(\mathcal{RS})$.

"The corresponding facit of the theory of cylindric algebras is to describe the cylindric set algebras $\mathfrak{Cs}(\mathfrak{M})$ for important models $\mathfrak{M}$. This amounts to looking at complete theories only, which is customary in model theory. It is somewhat surprising that this aspect of the theory of cylindric algebras has been almost entirely neglected. A complete description of $\mathfrak{Cs}(\mathfrak{M})$ is known only in the case in which $\mathfrak{M}$ has only one-place relations. There are many other simple structures where the description of $\mathfrak{Cs}(\mathfrak{M})$ should not be difficult; for example, for $\mathfrak{M}$ the rationals under their natural ordering."

[*Monk, J.D.: An introduction to cylindric set algebras. Logic Journal of the IGPL 8, 451–496 (2000)*]

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Examples of concept algebras ...

Neat Reducts

$$\mathfrak{Nr}_n\mathfrak{Cs}(\mathfrak{M})$$

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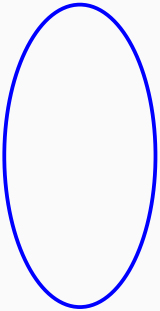
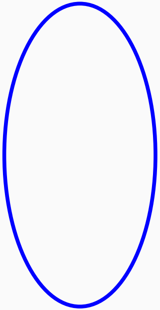
$$\mathfrak{Nr}_1\mathfrak{Cs}(\mathfrak{M}) \subseteq \cdots \subseteq \mathfrak{Nr}_n\mathfrak{Cs}(\mathfrak{M}) \subseteq \cdots$$

Homogeneous Structures



Homogeneous structure $\mathfrak{M}$

Homogeneous Structures



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The neat reduct $\mathfrak{Nr}_n\mathfrak{Cs}(\mathfrak{M})$ is atomic (in fact, finite)!

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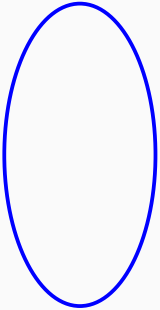

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$$\epsilon(\sim) \wedge \varphi$$

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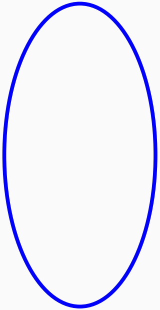

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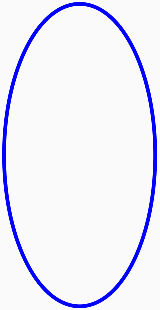

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v_1 v_0

v_2 v_3

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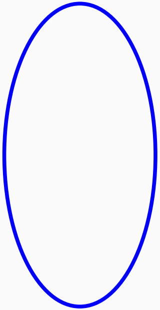

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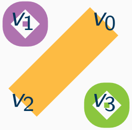
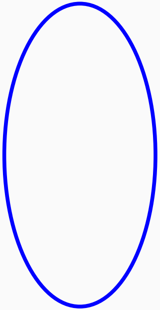


Equivalence relation $\sim$

Homogeneous Structures



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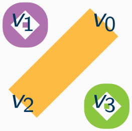

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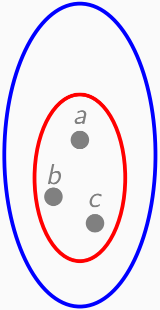

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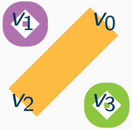

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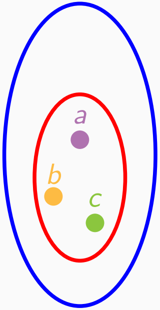

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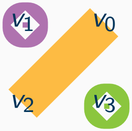

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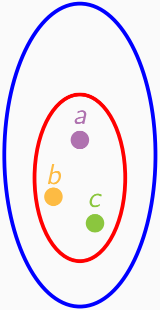

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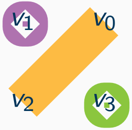

Equivalence relation $\sim$

$$\varphi \quad : \quad \begin{array}{l} v_1 \mapsto a \\ v_2 \mapsto b \\ v_3 \mapsto c \end{array}$$

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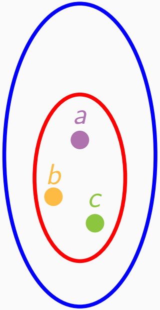

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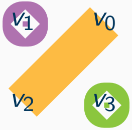

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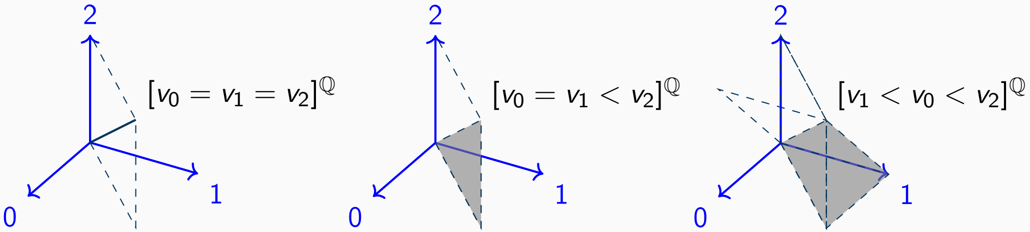

The 3-dimensional atoms of $\mathfrak{Cs}\mathbb{Q}$ up to symmetries of indices

Non-homogeneous structures

$\mathfrak{M}$ not homogeneous!

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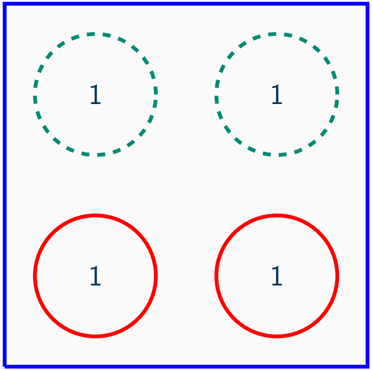

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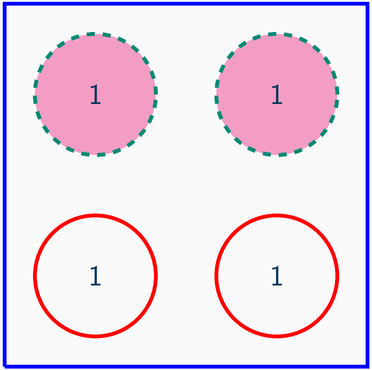

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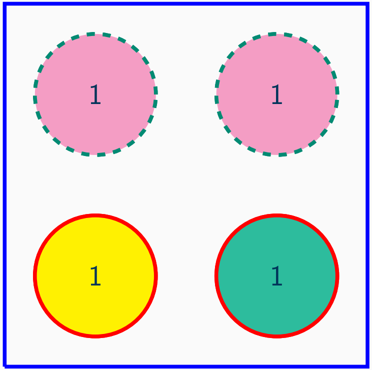

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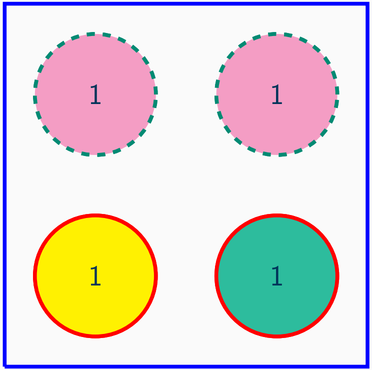
$R_{\bullet\bullet}$



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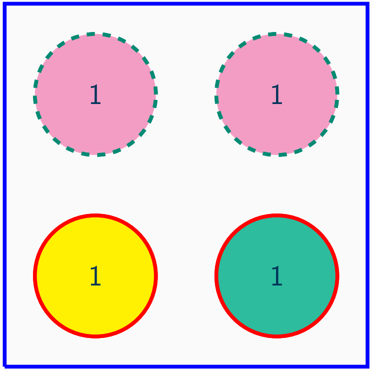

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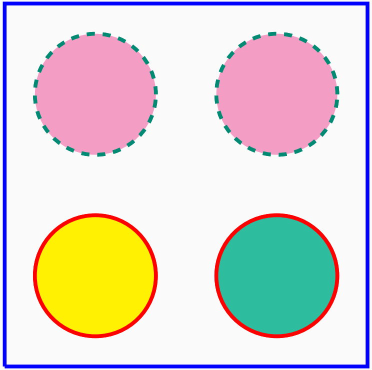

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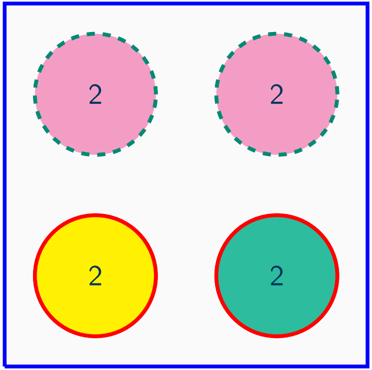

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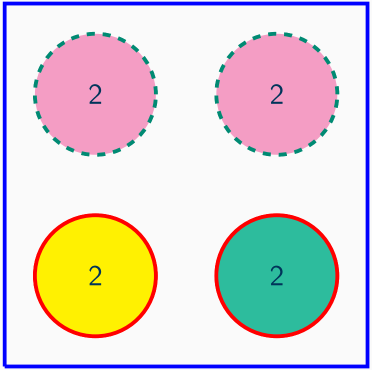
R

$$\mathfrak{M} \longrightarrow \mathfrak{M}_1^+ \longrightarrow \mathfrak{M}_2^+$$



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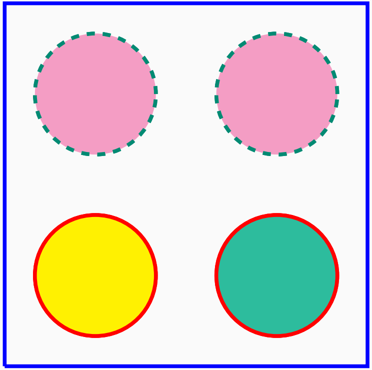

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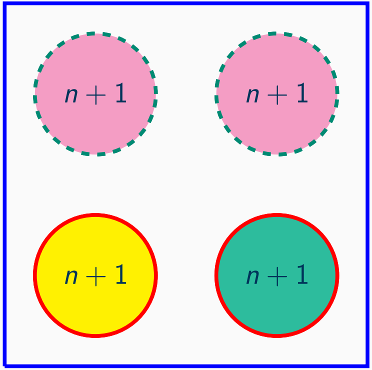

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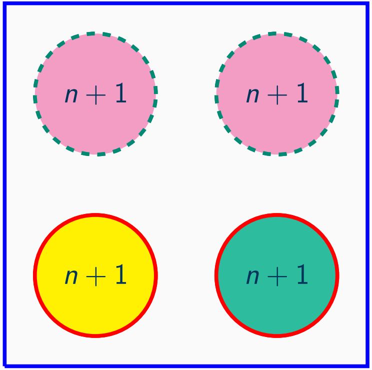

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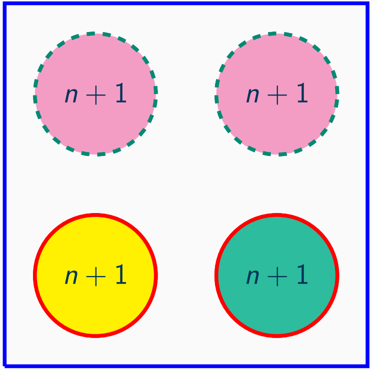

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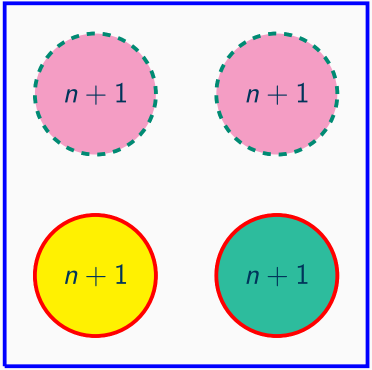

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Thank you!