

# WEAKENING RELATION ALGEBRAS

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ABSTRACT. In honor of Istvan Nemeti's 80th birthday, this talk will focus on algebras of binary relations that are slightly more general than representable relation algebras. Let  $\leq$  be a partial order on a set  $X$ . A binary relation  $R$  on  $X$  is a *weakening relation* with respect to  $\leq$  if  $\leq; R; \leq \subseteq R$ . Weakening relations have applications in sequent calculi, proximity lattices/spaces, order-enriched categories, intuitionistic modal logic, mathematical morphology and program semantics, e.g. via separation logic.

The set of all weakening relations on  $(X, \leq)$  is denoted by  $\mathbf{Wk}(X, \leq)$ , and it is closed under the operations of composition ( $;$ ), union ( $+$ ), intersection ( $\cdot$ ), complement-converse ( $\sim R = \neg R \sim$ ) and has  $\leq$  as identity element for composition. The algebra  $\mathbf{wk}(\leq) = (\mathbf{Wk}(X, \leq), +, \cdot, \emptyset, \top, ;, \sim, 1)$ , where  $\top = X^2$  and  $1 = \leq$ , is called a *full representable weakening relation algebra*, and the class of all subalgebras of products of such algebras is denoted by  $\mathbf{RwkRA}$ . The set  $\mathbf{Wk}(X, \leq)$  is also closed under a Heyting implication

$$R \rightarrow S = \{(x, y) \mid \forall u, v (u \leq x \ \& \ y \leq v \ \& \ uRv \implies uSv)\}$$

and if this operation is added then the resulting algebra, denoted  $\mathbf{Wk}(\leq)$ , is a cyclic involutive generalized bunched implication algebra and the SP-closure of this class is denoted by  $\mathbf{RWkRA}$ .

In this talk we will present what is currently known about  $\mathbf{RwkRA}$  and  $\mathbf{RWkRA}$ , and how they are related to the variety  $\mathbf{RRA}$  of representable relation algebras. The results about  $\mathbf{RWkRA}$  are based on joint research with Nick Galatos [1, 2] and the more recent results about  $\mathbf{RwkRA}$  are based on joint research with Jaš Šemrl.

## REFERENCES

- [1] N. Galatos and P. Jipsen, The structure of generalized BI-algebras and weakening relation algebras, *Algebra Universalis*, (2020) 81: 35, 1–35
- [2] N. Galatos and P. Jipsen, Weakening relation algebras and  $\mathbf{FL}^2$ -algebras, in *proc. 18th RAM-iCS conference, LNCS Vol 12062, Springer (2020)*, 117–133