Happy birthday, István!

What gift?

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No grand theorem, alas!

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What gift?

No grand theorem, alas!

A proposal to make your work more widely applicable in philosophy of science.

A unified view of theories

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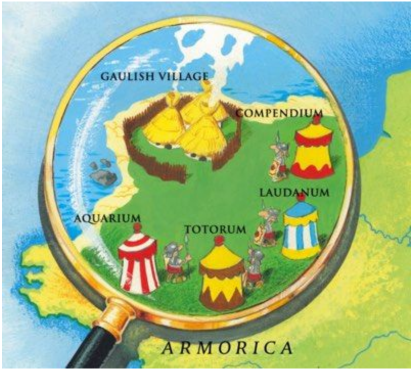
A logical perspective on theories

Long tradition:

- Hilbert, Frege, Gödel, Tarski, . . .
- Carnap, Quine, Putnam, David Lewis, . . .
- Ivstán and Hájnal
- Most of us here

Challenge

Logic-based approaches to analysing theories have become somewhat unpopular among philosophers of science.



Challenge

Hardly any actual physical theories have been represented in a logic-based way.

First response

István & Hájnal's great gift

Special relativity and some related theories represented in FOL.

Objection

Yes, but this is a special case: space-time geometry.

Hans (2016):

> *"And it is not at all clear that other interesting scientific theories could be reconstructed in this way – not even Einstein's general theory of relativity, nor quantum mechanics"*

We lack logic-based representations of run-off-the-mill theories that are formulated in terms of diff eq.

Second response

There is a logic-based way to represent diff eq theories.

Overview

Diagnosis

Unified approach

Conclusions and outlook

Syntactic view

Represent a theory's formalism as a set of sentences in FOL/HOL.

Suppes' diagnosis

The problem of background presuppositions:

> *"Almost all systematic scientific theories of any interest or power assume a great deal of mathematics as part of their formal background. There is no simple or elegant way to include this mathematical background in a standard formalization that assumes only the apparatus of elementary logic. This single point has been responsible for the lack of contact between much of the discussion of the structure of scientific theories by philosophers of science and the standard scientific discussions of these theories." (Suppes, 2002)*

Carnap's approach

Carnap (1939): a theory's formalism consists of

1. a formalised mathematical theory (the 'basic calculus'),
2. the specific formulas of the empirical theory.

Think: ZFC + EQ, type theory + EQ

Mismatch!

Model-solution correspondence

There should be a natural one-to-one correspondence between

- the models of the formalisation and
- the solutions as defined in scientific practice.

Mismatch between models and solutions.

- Logic: $(X, \in, \ldots, M, g, T)$
- Physics: (M, g, T)

Reason: non-standard models of basic calculus.

Putnam's point

We cannot get rid of unintended models by adding more object-language sentences to our formalisation.

Semantic presuppositions cannot be made explicit in the object-language.

Consequence
The syntactic view doesn't work
(if model-solution correspondence is desired).

Radical response

Semantic view of theories

- Let's not use object-language formulas as part of representation.
- Represent formalism as a class of mathematical structures (solutions/models as defined in practice).

NB: Trivially satisfies model-solutions correspondence.

Too radical

Hans (2012, 2013)

Not a good idea!

- We lose rigorous conceptual resources (definability, Morita/definitional equivalence, translations, etc.)
- What about theories without (known) models?

Lessons

1. Merely a set of object-language formulas will not suffice (if model-solution correspondence is desired).
2. Getting rid of the object-language is not a good idea either.

Proposal: unification of syntactic and semantic view of theories.

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Basic idea

A theory's formalism consists of

1. background framework: incl. family of structures
2. core: formulas in object language

Baby example:

- Framework: class of diff'able curves (over base structure)
- Core: differential equation singling out exponential ones

Frameworks

Definition

$(L, \Sigma, \mathcal{F}, Ad)$ is a framework if and only if

1. L is a logic,
2. Σ is a signature of L, partitioned into:
   Σ_F (fixed part), Σ_V (variable part),
3. $\mathcal{F}$ is a Σ_F-structure of L.
4. Ad is a (potentially structured) class of Σ-structures of L such that for every $\mathcal{A} \in Ad$: $\mathcal{A}|_{\Sigma_F} = \mathcal{F}$.

Example: a framework for QM

Logic: HOL

Σ comprises:

1. basic mathematical symbols: $\mathbb{R}$, $+$, $\cdot$, and definable vocabulary;
2. Hilbert space symbols:
   - a sort symbol H,
   - function symbols $\oplus : H \times H \to H$ and $\odot : \mathbb{C} \times H \to H$
   - a function symbol $\langle \cdot, \cdot \rangle : H \times H \to \mathbb{C}$
3. a function symbol $\hat{H} : H \to H$;
4. a function symbol $|\psi(\cdot)\rangle : \mathbb{R} \to H$;
5. an individual constant symbol $\hbar : \mathbb{R}$;
6. further HOL-definable vocabulary for convenience.

Σ_F: 1., 5. Σ_V: the rest

Example: a framework for QM

$\mathcal{F}$: standard model of Σ_F.

Ad is the class of Σ-structures $\mathcal{A}$ expanding $\mathcal{F}$ such that:

1. $\mathcal{H}$ is a separable complex Hilbert space,
   where $\mathcal{H}$ is the restriction of $\mathcal{A}$ to the Hilbert space symbols,
2. $\hat{H}^{\mathcal{A}}$ is a self-adjoint operator,
3. $|\psi(\cdot)\rangle^{\mathcal{A}}$ is a differentiable function from $\mathbb{R}^{\mathcal{A}}$ to $H^{\mathcal{A}}$.

Observation

The background framework typically encodes presuppositions, including semantic constraints.

Formalisms

Definition

(B, C) is a theoretical formalism if and only if

1. B is a framework and
2. C is a set of sentences over the signature of the framework B. (core formulas)

Let $T = (B, C)$ be a theoretical formalism.

- $Sol(T) := \{\mathcal{A}|_{\Sigma_V} : exists\ \mathcal{A} \in Ad(B)\ s.t.\ \mathcal{A} \models C\}$
- A Σ_V-assignment $\mathcal{A}$ satisfies a Σ-sentence φ in T iff $\mathcal{F} \cup \mathcal{A} \models \varphi$.

Example: a formalism for QM

Formalism QM consists of

1. the framework specified above,
2. core formula: the Schrödinger equation.

$$\forall_{t \in T} \quad i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

Sol(QM): solutions of the Schrödinger equation for all kinds of Hamiltonians on separable complex Hilbert spaces.

Customise as desired!
E.g. keep Hilbert space and Hamiltonian fixed.

Observation

The unified approach allows us to satisfy the model-solution correspondence principle.

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Conclusion

One can represent diff eq theories in a logic-based way.

Outlook

- Logical analysis of diff eq theories.
- Eternal peace between syntactic and semantic camps in philosophy of science.