Temporal Logic of Minkowski spacetime

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Dialogue

Me: how many spatial dimensions do we have?

You: three.

Me: how do you know its not less than three?

You: because I can make a regular tetrahedron where all sides have equal lengths, I couldn't do this with only two spatial dimensions.

Me: OK, I'm convinced. So can you write down a temporal formula which holds with three spatial dimensions but not with only two spatial dimensions?

You: let me think about that.

Abstract

According to relativity theory, the world is made up of space-time points which can send signals to each other at up to and including the speed of light. One striking difference with a Galilean model, is that there is no notion of simultaneity in relativity theory. So the Kripke frame where the worlds are space-time points and the accessibility is 'can send a signal to', branches densely in the future and in the past.

For special special relativity the Kripke frame is $(R^{n+1}, <)$ with one time dimension and n spatial dimensions, where two spacetime points are ordered by $<$ if and only if it is possible to send a signal from the first spacetime point to the second. For fixed n, there are four cases to distinguish according to whether this ordering is reflexive or irreflexive and whether signals may be sent at up to the speed of light, or strictly less than the speed of light. Temporal propositional formulas are built from propositions with propositional connectives and temporal operators F, P, G, H (sometime in the future, past, always in the future, past).

For each frame $(R^{n+1}, <)$ (where $<$ is reflexive/irreflexive, signals can/cannot go at speed of light, no. of spatial dimensions n = 1, 2, 3, ...) we consider three problems.

1. Distinguish these frames from each other by temporal formulas.
2. Find an axiomatisation of the temporal validities over each frame.
3. For each frame, is the validity problem for temporal formulas over the frame decidable, what is the complexity?

Very limited progress has been made with problems 1 and 2. We do have a few results for problem 3 (when n=1 the complexity is PSPACE complete, when $n > 1$ the complexity is EXPTIME hard, whether $<$ is reflexive or irreflexive, whether speed of light signals are allowed or not).