# Where Does General Relativity Break Down?

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conceptual puzzles, such as singularities. Will those problems be resolved by new high-energy physics?



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**Sagredo**: That's great! My favorite theory, general relativity, faces conceptual puzzles, such as singularities. Will those problems be resolved by new high-energy physics?

Salviati: Yes. We should expect general relativity to break down at high energies and be replaced by a quantum theory of gravity.



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1. Can we anticipate where our current theories will fail, and where they will be approximately or effectively correct? **Not as well as we think.** 

2. Can we take problems in general relativity to be "high energy" problems that will be resolved by a UV successor? **It depends.** 



#### Talk Overview



What is the "high energy regime" in general relativity?



Does general relativity fail (only) at high energies?



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#### Talk Overview



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One answer: high speed collisions produce a lot of energy; general relativity will break down as (part) of the description of such collisions. More generally: large stress-energy.

Another answer: general relativity should break down in the presence of large **gravitational energy**. **Uh oh...** 

(Non-local and quasi-local gravitational energy do not help: general relativity should break down locally and independently of background structures.)



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Rejoinder: But curvature is a tensor!



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Reply: Fine. Large **curvature scalars** indicate high energies.



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- Curvature measures tidal forces; large tidal forces reflect the "strong field" regime.
- 2 The Ricci scalar appears in the Einstein-Hilbert action; higher order terms will involve other scalar curvature quantities.
- 3 Curvature scalars measure energy density in other theories, such as electromagnetism.





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There exist singular solutions (e.g. plane waves) of Einstein's equation in which curvature "becomes large" but all curvature scalars vanish! (Cf. **??**)



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EM analogue: In flat spacetime,  $F_{ab} = f(u)x_{[a}\nabla_{b]}u)$ , where  $\nabla_a u$  is null and constant,  $x_a$  is constant, and  $x^a\nabla_a u = \mathbf{0}$ . Then stress-energy vanishes, as do all scalars constructed from  $F_{ab}$  and its derivatives—even if f(u) is singular.





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But there does not seem to be a single, scalar quantity whose large value always and unambiguously signals this regime.

This apparently means we cannot set a scale by a (scalar) cutoff, such that general relativity breaks down (only) when curvature scalars approach this value.



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#### Talk Overview

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Suppose we take singularities to signal a failure of general relativity.



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Image: A matrix

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Observe: there exist space-times that are flat, inextendible, and geodesically incomplete (?).



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Such spacetimes are "singular" but have vanishing curvature (and curvature scalars). "High energy physics" is not obviously relevant.

Stipulate: only curvature singularities (broadly construed) are "physical"; then it is plausible to think they will be resolved by quantum gravity.



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#### The real problem



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I contend: **Cauchy horizons** are just as troubling as singularities, and for similar reasons.



# Cauchy horizons

A Cauchy horizon is a boundary of the domain of validity of an initial value problem; initial data cannot be evolved past a Cauchy horizon.



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Cauchy horizons reflect a failure of the laws of physics to determine, or generate, future states from past ones.



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A Cauchy horizon is a boundary of the domain of validity of an initial value problem; initial data cannot be evolved past a Cauchy horizon.

Cauchy horizons reflect a failure of the laws of physics to determine, or generate, future states from past ones.

Some spacetimes with Cauchy horizons may be extended beyond the Cauchy horizon. But such extensions have a "global" character rather than a "local" one.



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Like singularities, Cauchy horizons are logically unrelated to curvature. There exist flat (extendible) spacetimes with Cauchy horizons.

Even in Kerr spacetime, curvature (and curvature scalars) are bounded in the vicinity of the Cauchy horizon.



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Question

Will quantum gravity resolve (physical) Cauchy horizons?



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Strong Cosmic Censorship

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#### Strong Cosmic Censorship

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Standard motivation: the SCCH says general relativity is (generically) **deterministic** in the sense of local Cauchy evolution.



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#### Strong Cosmic Censorship

**Generically**, the maximal Cauchy evolution of (suitable) initial data is (locally) inextendible.

Standard motivation: the SCCH says general relativity is (generically) **deterministic** in the sense of local Cauchy evolution.

Physical intuition (Penrose): Signals approaching a Cauchy horizon will be blue-shifted to arbitrarily high frequency near the horizon, generating (curvature) singularities.



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They claim: "it will follow that the  $C^0$ -inextendibility formulation of Penrose's celebrated strong cosmic censorship conjecture is in fact false" (abstract).



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Question

Is continuous extendibility what we should care about?



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The Dafermos and Luk result is nonetheless important.

It provides evidence **for** a more physically relevant SCCH: quantum gravity, insofar as it resolves (curvature) singularities will also generically resolve Cauchy horizons.



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## Two Points, Revisited

1. Can we anticipate where our current theories will fail, and where they will be approximately or effectively correct?

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1. Can we anticipate where our current theories will fail, and where they will be approximately or effectively correct? In general relativity, we probably cannot set a "scale" and say that if a fixed parameter approaches that scale, the theory breaks down; more generally, the theory can show pathological behavior even when curvature vanishes (or is small).

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2. Can we take problems in general relativity to be "high energy" problems that will be resolved by a UV successor? It depends on both the resolution of the (physical) strong cosmic censorship hypothesis, and also whether low-curvature pathologies can be eliminated by other considerations—such as kinematical constraints arising from quantum gravity.



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Thank you.1

<sup>1</sup>Based on work supported by the John Templeton Foundation. I am grateful to Sam Fletcher, Serge Rudaz, and Bob Wald for very helpful discussions in connection with this talk.

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Breakdown

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