Now, can or cannot classical kinematics interpret special relativity?

Gergely Székely

Rényi Institute

This talk is based on joint works with
Hajnal Andréka, Koen Lefever, Judit X. Madarász, and István Németi.

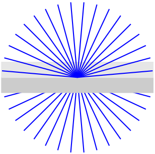
Overview

1. The non-interpretability of $Th(\mathcal{RS})$ in $Th(\mathcal{CS})$
2. The interpetation of SpecRel in ClassicalKin
3. Resolving the apparent contradiction
   - The key difference between ClassicalKin and $Th(\mathcal{CS})$
   - Hajnal Andréka's conjecture
   - Sketch of a missing bridge

The non-interpretability of $Th(\mathcal{RS})$ in $Th(\mathcal{CS})$

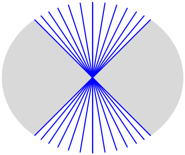
Classical **S**pacetime

$$\mathcal{CS} = \langle \mathbb{R}^4, \mathsf{col}^\infty \rangle$$



Relativistic **S**pacetime

$$\mathcal{RS} = \langle \mathbb{R}^4, \mathsf{col}^t \rangle$$



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Relativistic **S**pacetime

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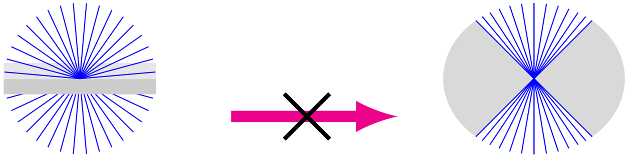

Theorem

$Th(\mathcal{CS})$ cannot be interpreted in $Th(\mathcal{RS})$.

Classical **S**pacetime

$$\mathcal{CS} = \langle \mathbb{R}^4, \mathsf{col}^\infty \rangle$$

Relativistic **S**pacetime

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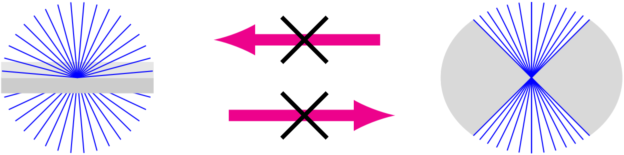


Theorem

$Th(\mathcal{CS})$ cannot be interpreted in $Th(\mathcal{RS})$.

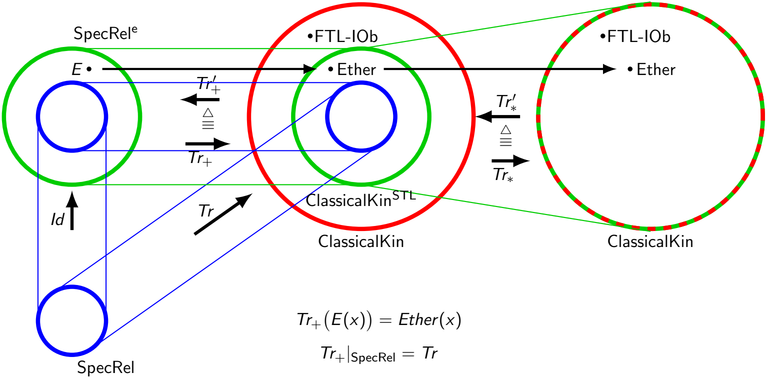
Theorem

$Th(\mathcal{RS})$ cannot be interpreted in $Th(\mathcal{CS})$, either.

The interpetation of SpecRel in ClassicalKin

Theorem

SpecRel^e *and* ClassicalKin *are definitionally equivalent.*

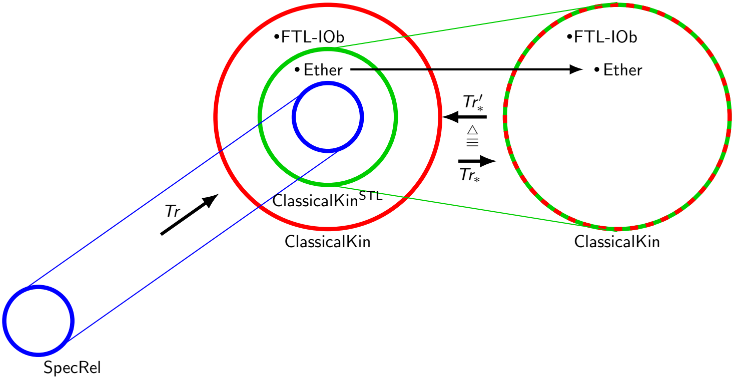


$$Tr_+(E(x)) = Ether(x)$$

$$Tr_+|_{\text{SpecRel}} = Tr$$

Theorem

SpecRel *can be interpreted in* ClassicalKin.



The paradox to resolve:

Theorem

$Th(\mathcal{RS})$ cannot be interpreted in $Th(\mathcal{CS})$.

Special relativity **cannot** be interpreted in classical kinematics.

Theorem

SpecRel *can be interpreted in* ClassicalKin.

Special relativity **can** be interpreted in classical kinematics.

So who is right?

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Everyone!

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How is that possible?

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$Th(\mathcal{RS}) \neq$ SpecRel and $Th(\mathcal{CS}) \neq$ ClassicalKin

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Yes, but... Shouldn't they be roughly/basically the same?

So who is right?

Everyone!

How is that possible?

$Th(\mathcal{RS}) \neq$ SpecRel and $Th(\mathcal{CS}) \neq$ ClassicalKin

Yes, but... Shouldn't they be roughly/basically the same?

Right, let's dig deeper!

Main differences between $Th(\mathcal{RS})$ and SpecRel:

- the language of SpecRel is more complex
- $\mathcal{RS}$ is scale-free
- SpecRel is not complete

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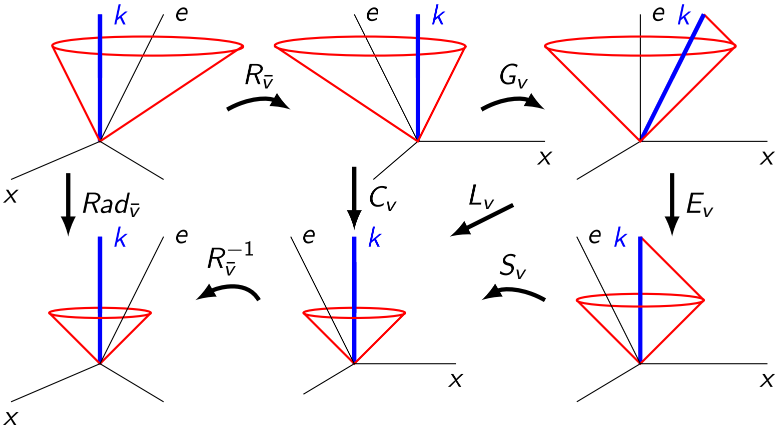
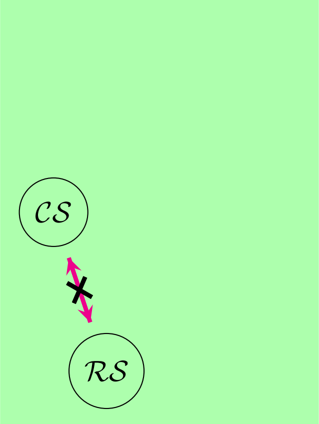
Similar differences between $Th(\mathcal{CS})$ and ClassicalKin:

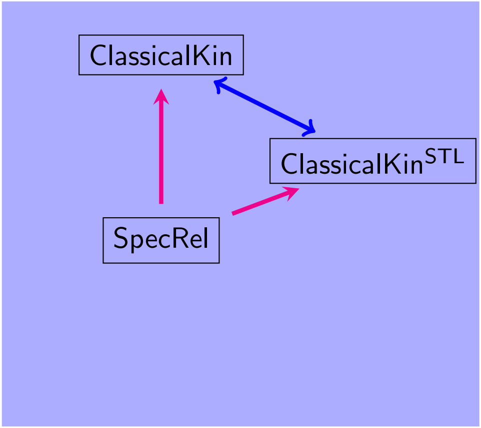
- the language of ClassicalKin is more complex
- $\mathcal{CS}$ is scale-free and ...
- ClassicalKin is not complete

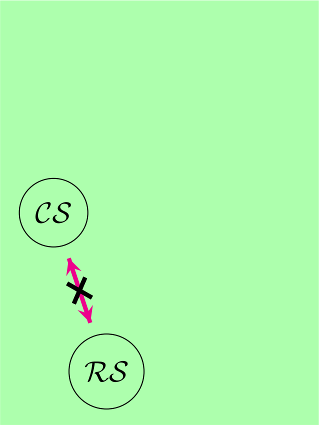
The key difference

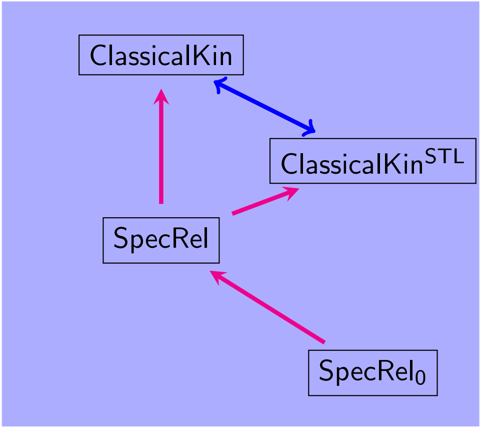


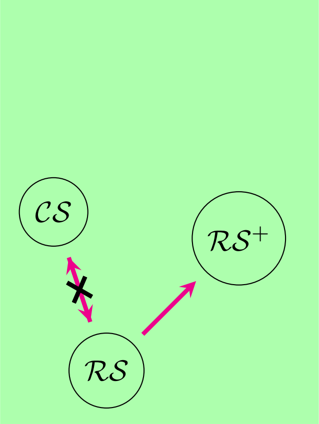
- There are light signals (of finite speed) in ClassicalKin.

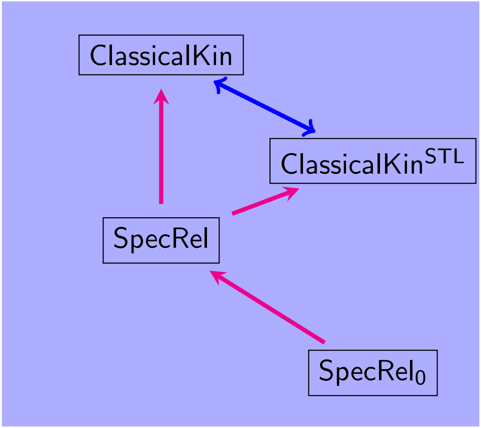

$\mathcal{CS}$
$\mathcal{RS}$


ClassicalKin
ClassicalKin$^{\mathsf{STL}}$
SpecRel


$\mathcal{CS}$
$\mathcal{RS}$

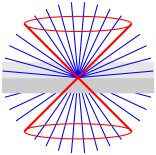

ClassicalKin
ClassicalKin^{STL}
SpecRel
SpecRel$_0$


$\mathcal{CS}$
$\mathcal{RS}^+$
$\mathcal{RS}$


ClassicalKin
ClassicalKin^{STL}
SpecRel
SpecRel$_0$

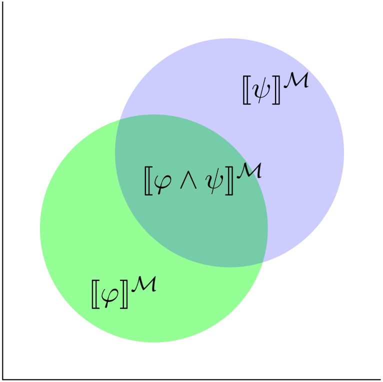
Classical spacetime, with light signals:

$$\mathcal{CS}^{\times} = \left\langle \mathbb{R}^4, \mathsf{col}^{\infty}, \mathsf{col}^{\lambda} \right\rangle$$

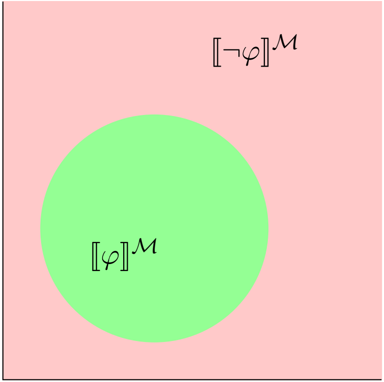


The **meaning** $[\![\varphi]\!]^{\mathcal{M}}$ of formula φ in model $\mathcal{M}$ is the set of sequences from $\mathcal{M}$ satisfying φ, i.e.

$$[\![\varphi]\!]^{\mathcal{M}} = \{\bar{a} \in M^{\omega} : \mathcal{M} \models \varphi[\bar{a}]\}\,.$$

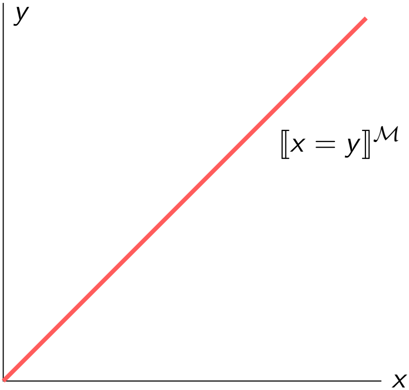


$$[\![\varphi \wedge \psi]\!]^{\mathcal{M}} = [\![\varphi]\!]^{\mathcal{M}} \cap [\![\psi]\!]^{\mathcal{M}}$$

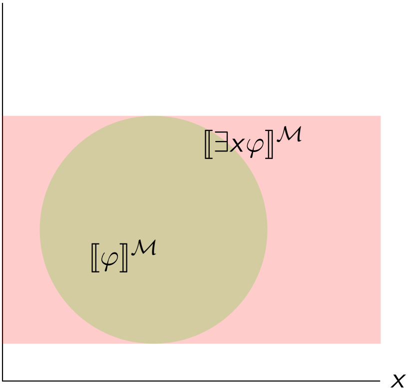


$$[\![\neg\varphi]\!]^{\mathcal{M}} = -[\![\varphi]\!]^{\mathcal{M}}$$

The **concept algebra** $\mathrm{CA}(\mathcal{M})$ of model $\mathcal{M}$ is a natural algebra of meanings of formulas in $\mathcal{M}$.



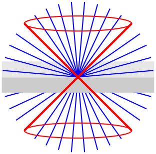
$[\![x = y]\!]^{\mathcal{M}} = D_{xy}$



$[\![\exists x\varphi]\!]^{\mathcal{M}} = C_x[\![\varphi]\!]^{\mathcal{M}}$

Classical spacetime, with light signals:

$$\mathcal{CS}^{\times} = \left\langle \mathbb{R}^4, \mathsf{col}^{\infty}, \mathsf{col}^{\lambda} \right\rangle$$



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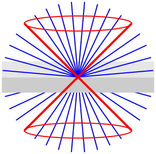

$\mathcal{RS}$ is definitionally equivalent to $\langle \mathbb{R}^4, \mathsf{col}^{\lambda} \rangle$. So the concept algebra $\mathsf{CA}(\mathcal{RS})$ is (isomorphic to) a subalgebra of $\mathsf{CA}(\mathcal{CS}^{\times})$.

Classical spacetime, with light signals:

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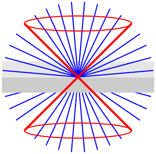
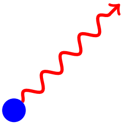


$\mathcal{RS}$ is definitionally equivalent to $\left\langle \mathbb{R}^4, \mathsf{col}^{\lambda} \right\rangle$. So the concept algebra $\mathsf{CA}(\mathcal{RS})$ is (isomorphic to) a subalgebra of $\mathsf{CA}(\mathcal{CS}^{\times})$.

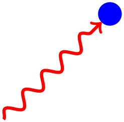
Conjecture of Hajnal Andréka, 2017

Every concept from $\mathsf{CA}(\mathcal{CS}^{\times})$ which is not already in the subalgebra $\mathsf{CA}(\mathcal{RS})$ generates together with $\mathsf{CA}(\mathcal{RS})$ the whole concept algebra $\mathsf{CA}(\mathcal{CS}^{\times})$.

James Ax (1978) $\rightsquigarrow$ SigTh
(a **Sig**naling **Th**eory of special relativity)



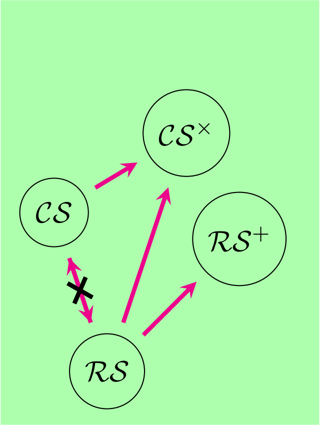
a particle sending out a singal



a particle receiving a singal

Theorem (Andréka–Németi, 2014)

SigTh *is definitionally equivalent to* $\mathsf{SpecRelCompl}_0^{\uparrow}$.


$\mathcal{CS}^\times$
$\mathcal{CS}$
$\mathcal{RS}^+$
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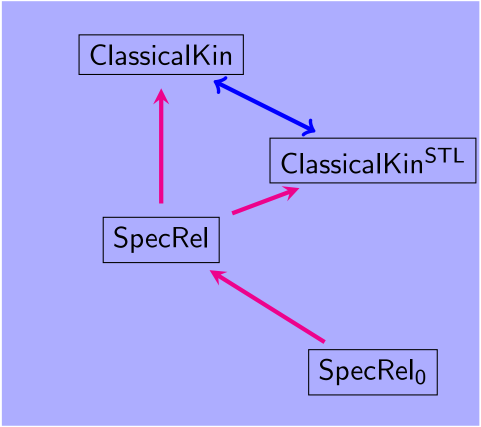
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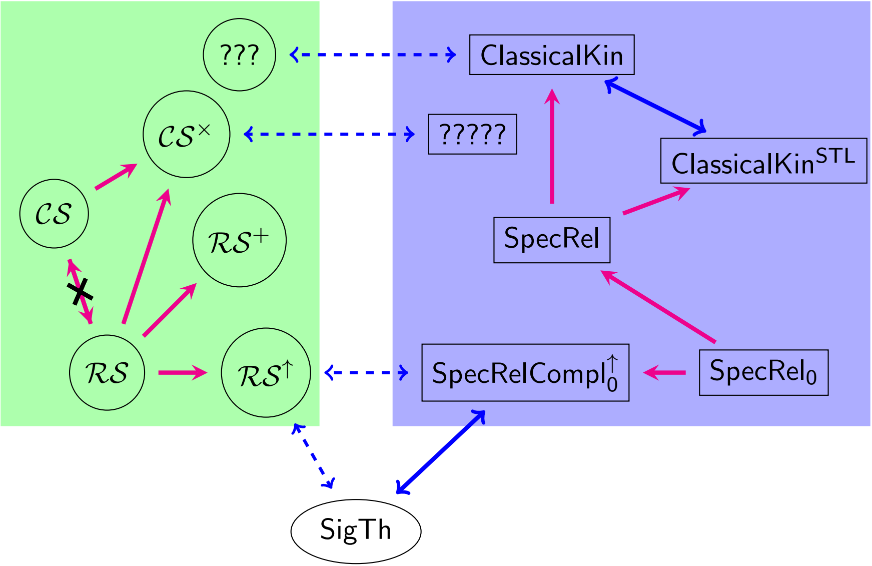
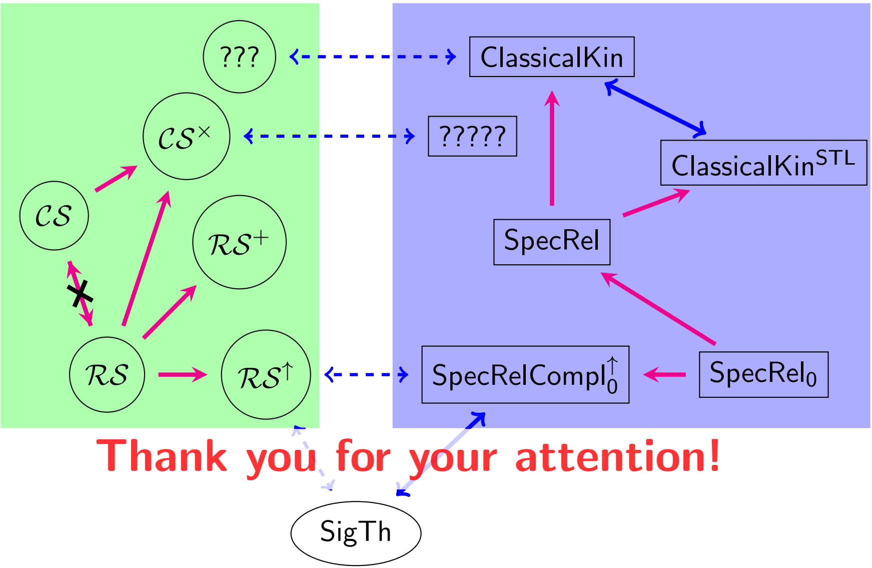

???
$\mathcal{CS}^{\times}$
$\mathcal{CS}$
$\mathcal{RS}^{+}$
$\mathcal{RS}$
$\mathcal{RS}^{\uparrow}$
ClassicalKin
?????
$\text{ClassicalKin}^{\text{STL}}$
SpecRel
$\text{SpecRelCompl}_0^{\uparrow}$
SpecRel_0
SigTh



Thank you for your attention!