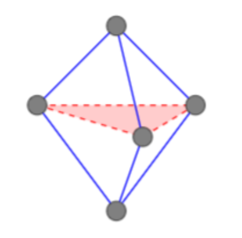

Concept Algebras and Conceptual Distance

Mohamed Khaled
Faculty of Engineering and Natural Sciences, Bahçeşehir University, Istanbul

Concept Algebras ...

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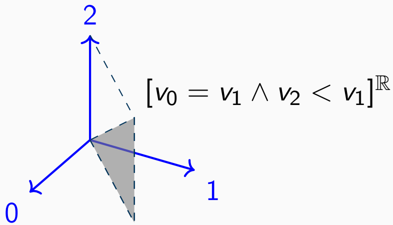

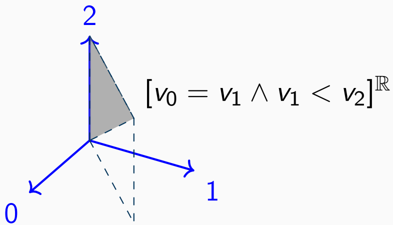

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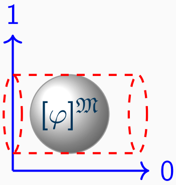

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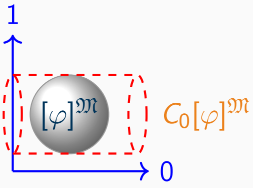

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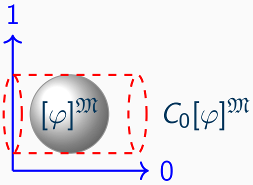

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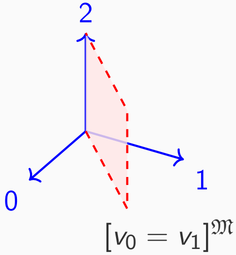

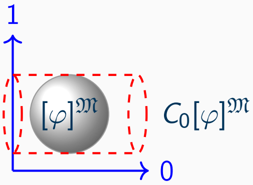

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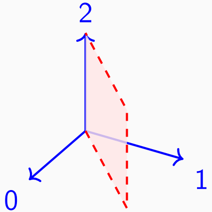

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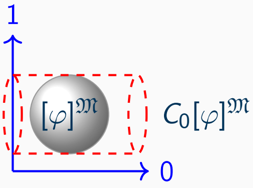

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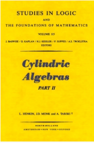
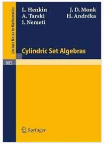
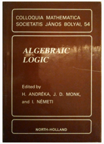
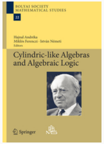
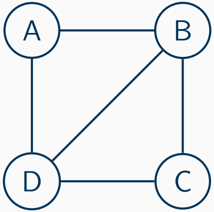








Ex: $\mathfrak{Cs}(G)$ of a simple graph G



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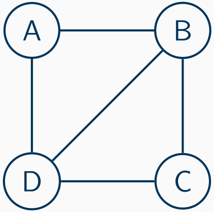

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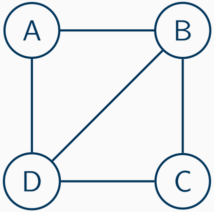

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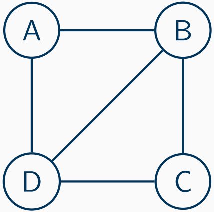

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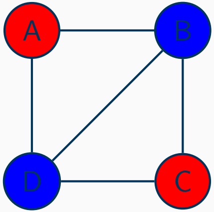

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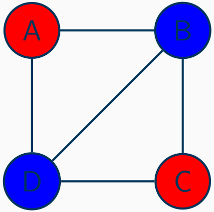

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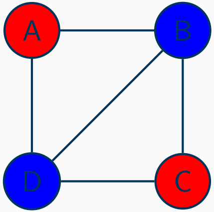

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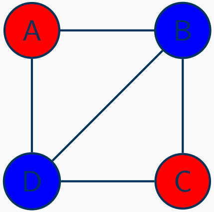

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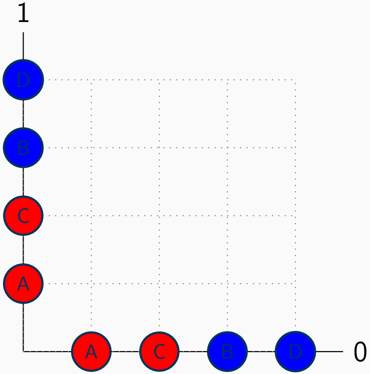

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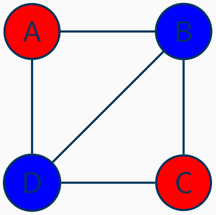

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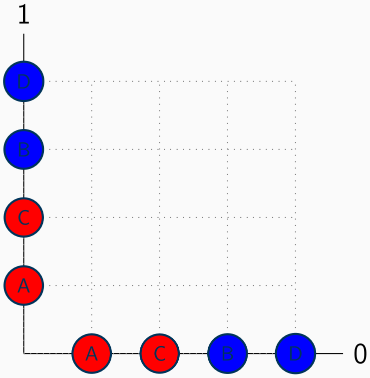

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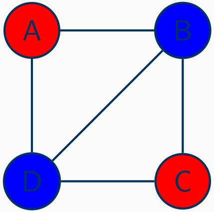

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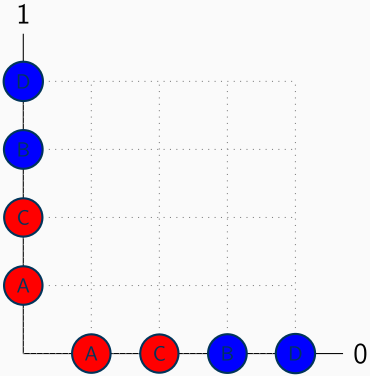

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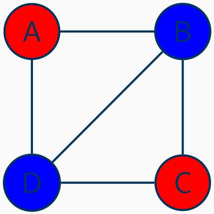

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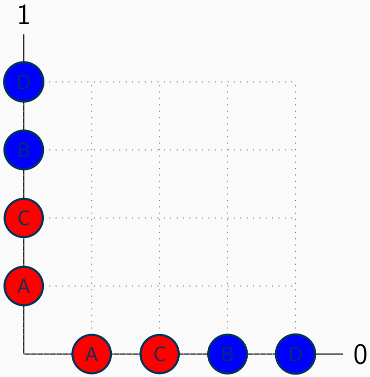

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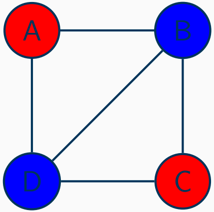

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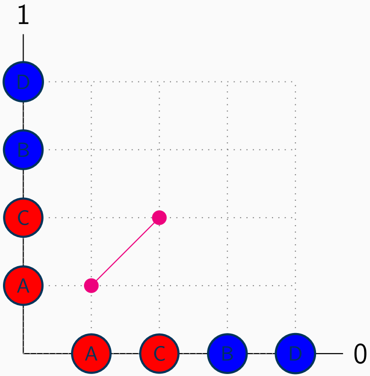

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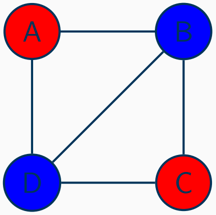

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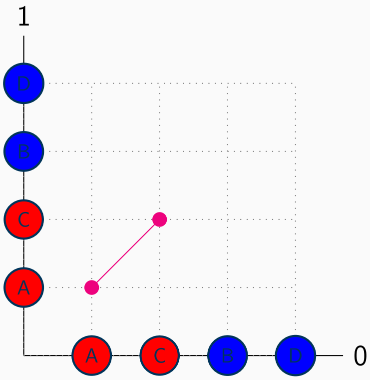

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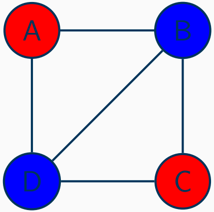

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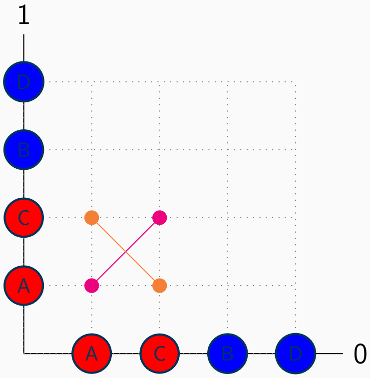

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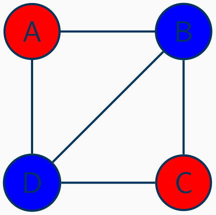

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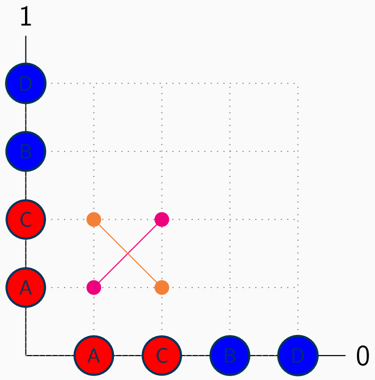

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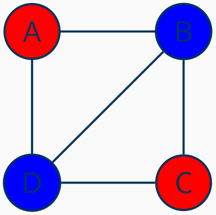

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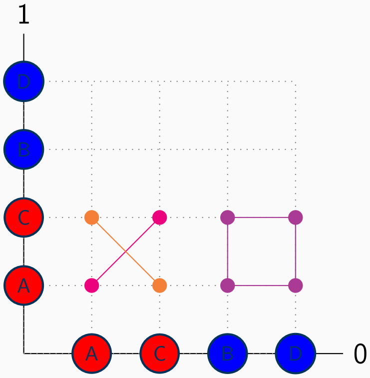

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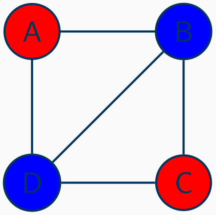

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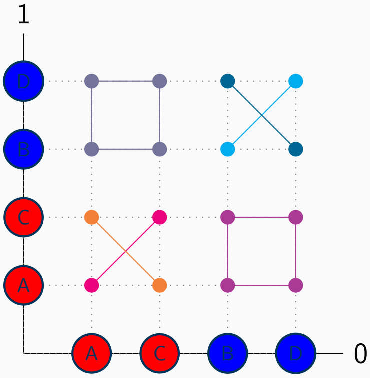

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"The corresponding facit of the theory of cylindric algebras is to describe the cylindric set algebras $\mathfrak{Cs}(\mathfrak{M})$ for important models $\mathfrak{M}$. This amounts to looking at complete theories only, which is customary in model theory. It is somewhat surprising that this aspect of the theory of cylindric algebras has been almost entirely neglected. A complete description of $\mathfrak{Cs}(\mathfrak{M})$ is known only in the case in which $\mathfrak{M}$ has only one-place relations. There are many other simple structures where the description of $\mathfrak{Cs}(\mathfrak{M})$ should not be difficult; for example, for $\mathfrak{M}$ the rationals under their natural ordering."

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Network of Cs ...

Logic vs Algebra I

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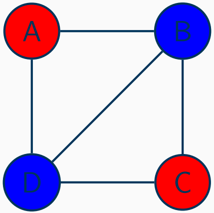
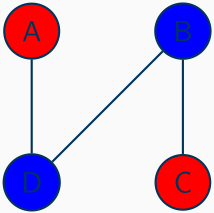
$$\mathfrak{M} \rightleftarrows \mathfrak{N} \iff \mathfrak{Cs}(\mathfrak{M}) \cong \mathfrak{Cs}(\mathfrak{M})$$

Logic vs Algebra I

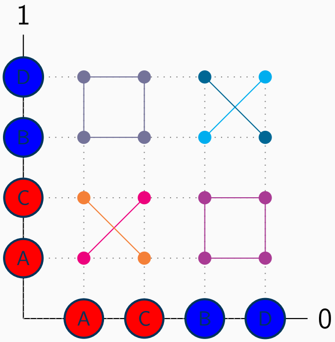


Simple graph G



Simple graph H

Logic vs Algebra I



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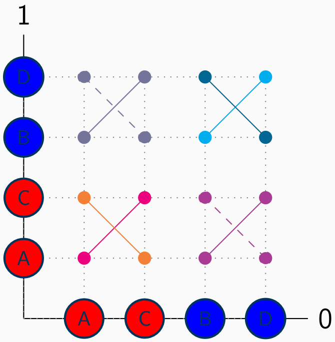
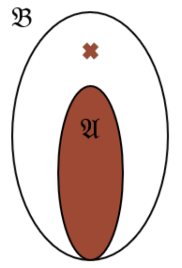


$\mathfrak{Nr}_2\mathfrak{Cs}(H)$

Logic vs Algebra II

Logic vs Algebra II

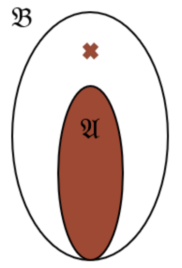


Logic vs Algebra II

Definition

$\mathfrak{Cs}(\mathfrak{M})$ is a **large subalgebra** of $\mathfrak{Cs}(\mathfrak{N})$:

$$\mathfrak{Cs}(\mathfrak{M}) \rightharpoonup \mathfrak{Cs}(\mathfrak{N}) \overset{\text{def}}{\iff} \exists a \in \mathfrak{Cs}(\mathfrak{N}) \text{ such that } \langle Cs(\mathfrak{M}) \cup \{a\} \rangle = \mathfrak{Cs}(\mathfrak{N})$$



Logic vs Algebra II

Definition

$\mathfrak{N}$ is a **one concept-extension** of $\mathfrak{M}$:

$$\mathfrak{M} \rightsquigarrow \mathfrak{N} \overset{\text{def}}{\iff} M = N, \quad \mathcal{L}(\mathfrak{N}) = \mathcal{L}(\mathfrak{M}) \cup \{R\} \quad \text{and} \quad \mathfrak{M} \sqsubseteq \mathfrak{N}$$

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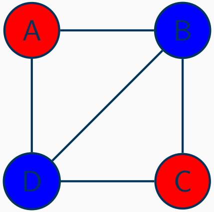
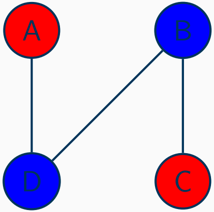
$$\exists\, \mathfrak{N}' \text{ such that } \mathfrak{M} \rightsquigarrow \mathfrak{N}' \rightleftarrows \mathfrak{N} \iff \exists\, \mathfrak{N}' \text{ such that } \mathfrak{Cs}(\mathfrak{M}) \rightharpoonup \mathfrak{Cs}(\mathfrak{N}') \cong \mathfrak{Cs}(\mathfrak{N})$$

Logic vs Algebra II



Simple graph G



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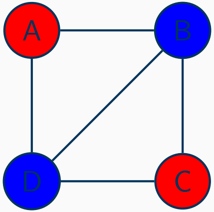
Logic vs Algebra II



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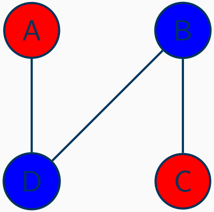
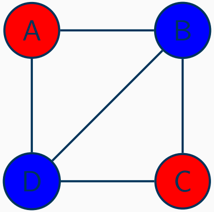


Simple graph H

$$\exists\ \mathfrak{N}' \text{ such that } G \rightsquigarrow \mathfrak{N}' \rightleftarrows H$$

Logic vs Algebra II



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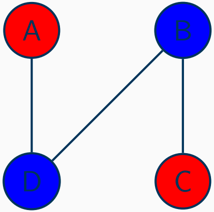

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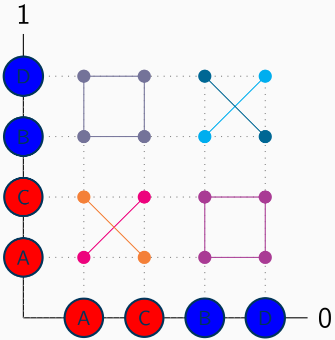
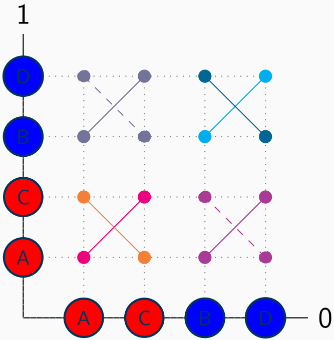


$\mathfrak{Nr}_2\mathfrak{Cs}(G)$



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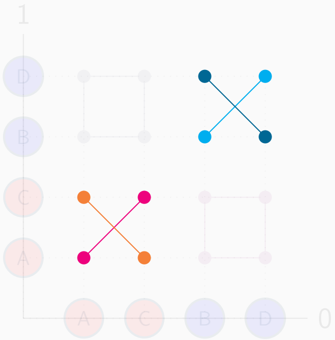

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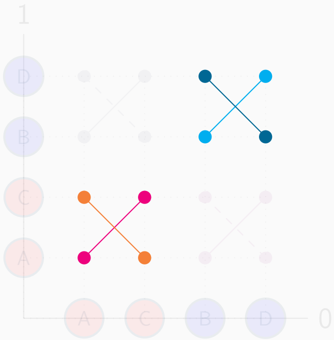

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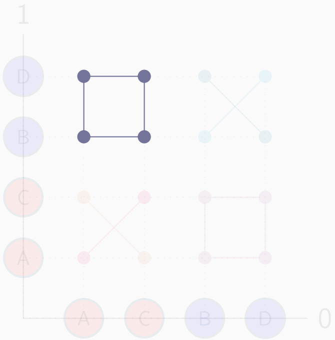

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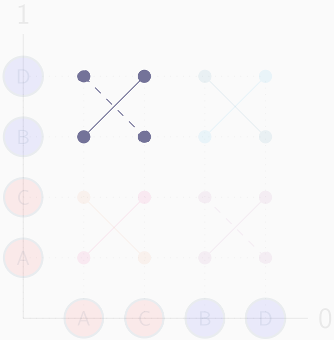

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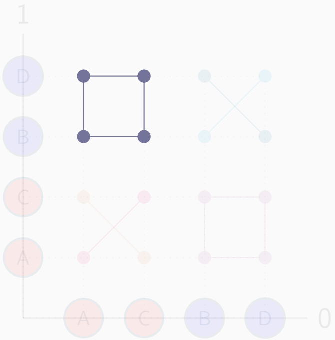

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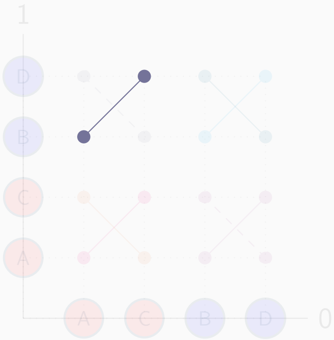

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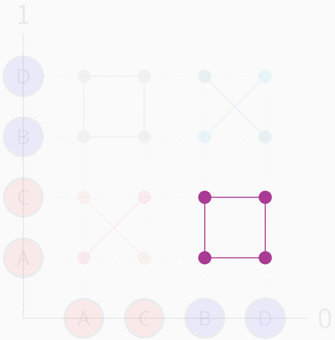

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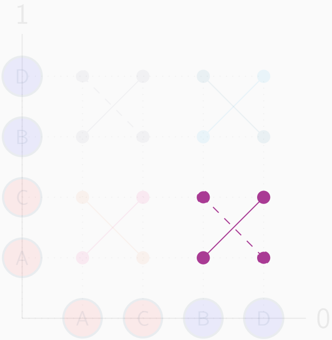

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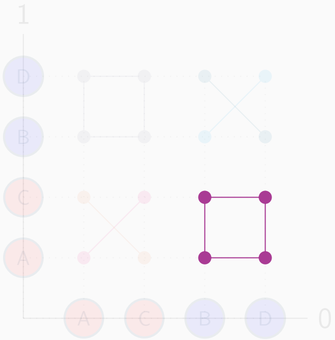

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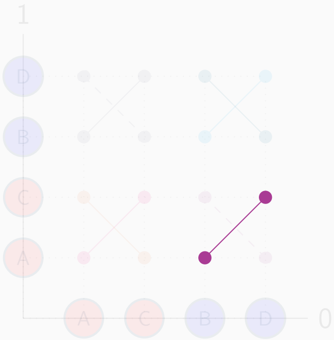

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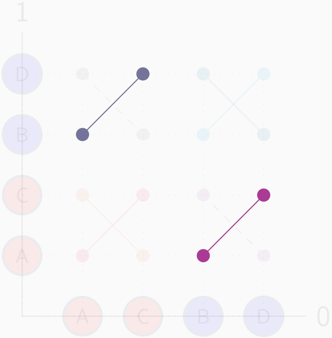
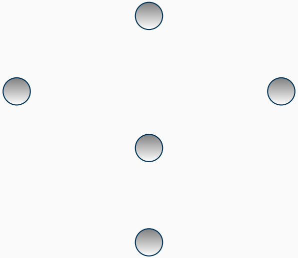


$\mathfrak{Nr}_2\mathfrak{Cs}(H)$

Network of concept algebras

Network of concept algebras

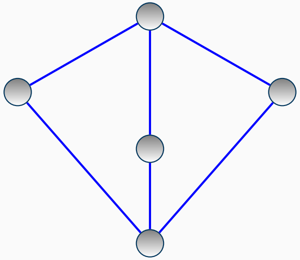
Nodes representing CAs



Network of concept algebras

Nodes representing CAs

Blue Edges: adjacent by a large inclusion

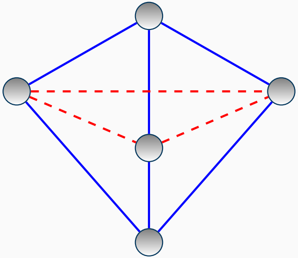


Network of concept algebras

Nodes representing CAs

Blue Edges: adjacent by a large inclusion

Dashed Red Edges: isomorphic algebras



Conceptual distance

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If $\mathfrak{A}$ and $\mathfrak{B}$ are not connected, then $d_{\mathfrak{g}}(\mathfrak{A}, \mathfrak{B}) \stackrel{\text{def}}{=} \infty$.

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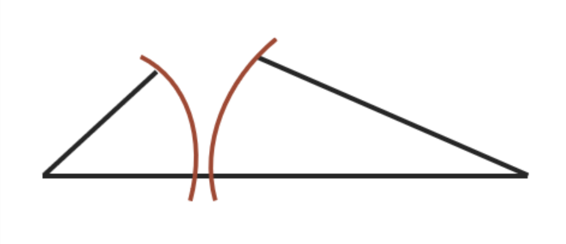

Theories of physics …

Theories of physics

Theories of physics

Hajnal Andréka Judit Madarász István Németi Gergely Székely

Theories of physics

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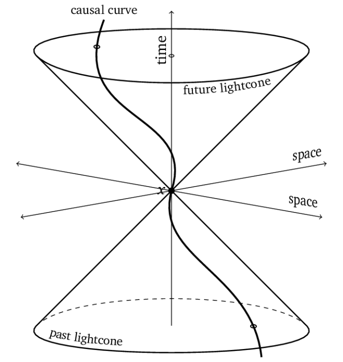
SpecRel



Theories of physics

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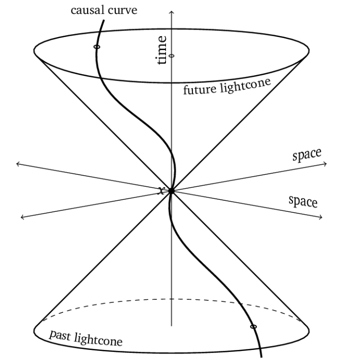
SpecRel

ClassicalKin



Lefever's PhD thesis

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Theorem (K. Lefever & G. Szekély 2017)

Classical and relativistic kinematics are distinguished by only one concept.

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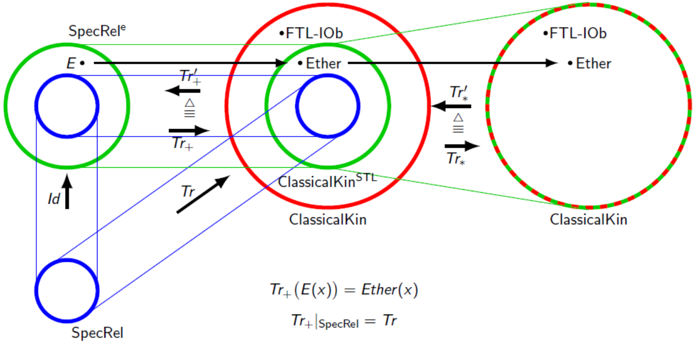

Comparison of theories

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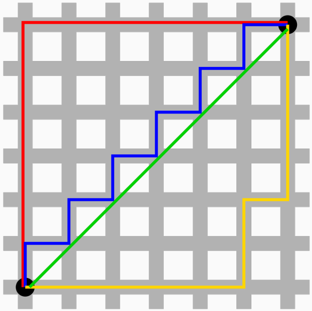
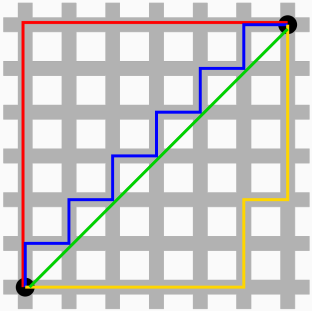


Metric Functions

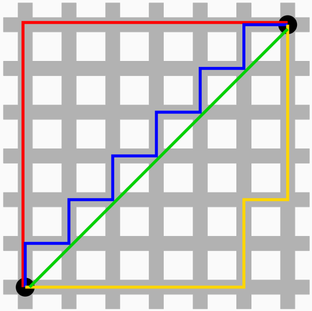
Comparison of theories



Metric Functions

- Precise definitions for concept adding/removal?

Comparison of theories



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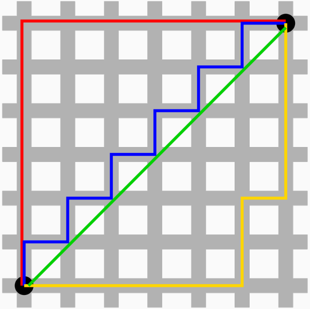

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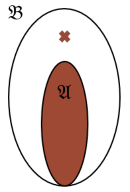
$$\mathfrak{A} \xrightarrow{\langle \bullet \rangle} \mathfrak{B}$$



More on the project

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- M. Khaled, G. Székely, K. Lefever and M. Friend (2020). DISTANCES BETWEEN FORMAL THEORIES. **The Review of Symbolic Logic**, 13(3), pp. 633 – 654.
- M. Khaled and G. Székely (2021). ALGEBRAS OF CONCEPTS AND THEIR NETWORKS. In: T. Allahviranloo, S. Salahshour, N. Arica (eds), **Progress in Intelligent Decision Science. IDS 2020. Advances in Intelligent Systems and Computing**, vol 1301. Springer, pp. 611–622.
- T. Aslan, M. Khaled and G. Székely (2021). ON THE NETWORKS OF LARGE EMBEDDINGS. **In preparation**.

Thank you!