# On Surplus Structure Arguments

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# Leibniz's Static & Kinematic Shift Arguments

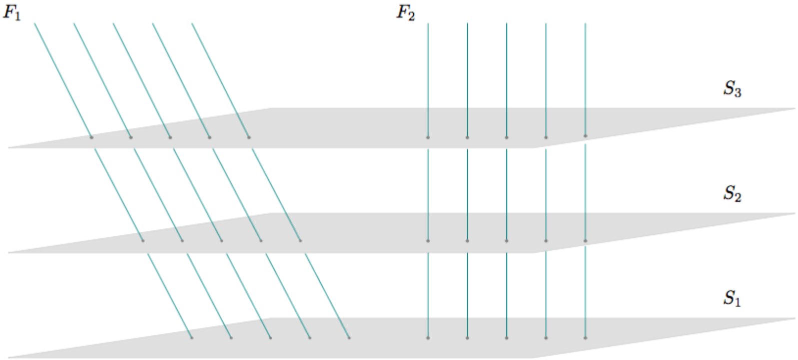


Fig. 5 of DiSalle, Robert, "Space and Time: Inertial Frames", The Stanford Encyclopedia of Philosophy (Summer 2020 Edition), Edward N. Zalta (ed.) `https://plato.stanford.edu/archives/sum2020/entries/spacetime-iframes/`

# Belot's Challenge

Surplus structure arguments seem to commit to the following two principles (p. 319):

**D1**: The symmetries of a classical theory are those transformations that map solutions of the theory's equation of motion to solutions of the theory's equation of motion.

**D2**: Two solutions of a classical theory's equation of motion are related by a symmetry if and only if they are physically equivalent . . .

$\Rightarrow$ All solutions are physically equivalent.

> "I leave it as a challenge to the reader to identify a general and interesting formal notion of symmetry [i.e., a rendering of D1] that renders D2 true" (p. 333).

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# General Questions

1. How should symmetries be defined?
   - More or less formally, with different *types* for what structures they preserve.
2. What is the relationship between symmetry and interpretation/physical equivalence?
   - The surplus structure argument gives a valid *ceteris paribus* argument.
   - This argument only invokes certain specific types of symmetry.

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# Outline

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# Models and Properties

$\mathcal{S}$: a space of models/states of a physical system characterizing some phenomenon of interest.

$X$: a property assignment to (at least some of) $\mathcal{S}$. E.g.,

- $X : \mathcal{S} \to \{\bot, \top\}$ represents a Boolean property.
- $X : \mathcal{S} \to \mathbb{R}$ represents a real-valued property.

## Example

The Boolean property $L$ picks out the *nomologically possible* models, i.e., the ones that satisfy the *laws* of a theory of interest.

# Observable Properties

Some property assignments $O$ represents observable properties. (Not a merely formal matter.)

$\sim_O$: equivalence relation on $\mathrm{cod}(O)$ of *mutual observational indistinguishability*.

$\sim$: equivalence relation on $\mathcal{S}$ of mutual observational indistinguishability: $s \sim s'$ iff $O(s) \sim_O O(s')$ for all observable properties $O$.

# Transformation and Preservation

Let a bijection $T : \mathcal{S} \to \mathcal{S}$ that is not an automorphism be called a *transformation* of $\mathcal{S}$.

Some transformations *preserve* properties, relations, or other structures on $\mathcal{S}$.

### $X$-symmetry

$T$ preserves the assignment $X$ iff for all $s \in \mathcal{S}$, $X(s) = (X \circ T)(s)$.

# Transformation and Observation

## Observational Symmetry

$T$ preserves observational properties iff for all $s \in \mathcal{S}$, $s \sim T(s)$.

Note that this is stronger that preserving mutual observational distinguishability since it requires $O = O \circ T$ for all observational properties $O$.

# Outline

# The Surplus Structure Argument

Let $\mathcal{S}$ be the kinematically possible models and $L$ be the Boolean property assignment of nomological possibility.

1. Suppose that $X$ is a property assignment and $T$ is a transformation that is an observational symmetry *and* an $L$-symmetry but *not* an $X$-symmetry.
2. Thus, $X$ represents a property that is not observationally detectable.
3. Ceteris paribus, we should prefer an interpretation of $\mathcal{S}$ that dispenses with (the reality of) $X$ over one that does not.

There are two (related) Occamist norms in play: ontological and representational.

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## Why Ceteris Paribus?

- The Occamist norms censure one theoretical vice among many.
- We may not have a suitable alternative.
- Intertheoretic relations, such as embedding and unity of interpretation.

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# Vicious Circularity?

A surplus structure argument presupposes a list of observational properties, yet

> *we often use premises about symmetries in order to work out which physical features fix the [observational] data, so we cannot at the same time define symmetries to be those operations that preserve features that fix the [observational] data (p. 865)*

My response:

- What matters is the coherence of an interpretation with symmetry.
- Employ reflective equilibrium: 4 steps.

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# Reflective Equilibrium I

Propose plausible and empirically adequate (or nearly enough so) theory of a system of interest:

- states or models
- observable properties
- representation relations

Then, identify the observational symmetries.

## Reflective Equilibrium II

Test both the models, observable properties, and representation relations:

### Wide enough?

- Test by experimentation.
- Needs to account for plethora of experience.
- Can introduce new states or observables.

### Narrow enough?

- Test by surplus structure arguments.
- Dispense with properties that add little value to the theory.
- Can reduce states or observables.

Eliminating states or observable properties can introduce more observational symmetries, while introducing new states or observables can eliminate some of them.

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## Reflective Equilibrium III

If surplus structure arguments identify surplus structure, one can:

- quotient states by an observational symmetry to eliminate it, or
- change some observational symmetries to isomporphisms.

Which technique one chooses depends on balancing the interpretative and explanatory considerations at hand.

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# Reflective Equilibrium IV

Return to step II as new theoretical arguments and experimental facts about observables and their representation arises.

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# Back to Belot

Contra Belot, neither of his two principles play a role in the surplus structure argument (SSA):

**D1**: The symmetries of a classical theory are those transformations that map solutions of the theory's equation of motion to solutions of the theory's equation of motion.

- $L$-symmetries are just one type, and those that figure in the SSA are also observational symmetries.

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1. The SSA provides a link between interpretation and symmetry and has a valid ceteris paribus form using Occamist norms.
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