Online Conference in Automorphic Forms 2020
Abstracts

2020 June 1-5

Balkanova, Olga (Steklov Mathematical Institute): Spectral decomposition formula and moments of symmetric square L-functions

We prove a spectral decomposition formula for averages of Zagier L-series

$$\sum_{l=1}^{\infty} \omega(l) \mathcal{L}_{n^2-4l^2}(s)$$

in terms of moments of symmetric square L-functions associated to Maass and holomorphic cusp forms of levels 4, 16, 64.

Blomer, Valentin (University of Bonn): A symplectic restriction problem

We investigate the norm of a degree 2 Siegel modular form of large weight whose argument is restricted to the 3-dimensional subspace of its imaginary part. On average over Saito–Kurokawa lifts an asymptotic formula is established that supports both the mass equidistribution conjecture on the Siegel upper half space as well as the Lindelöf hypothesis for the corresponding Koecher–Maaß series. The ingredients include a new relative trace formula for pairs of Heegner periods. This is joint work with Andrew Corbett.

Bringmann, Kathrin (University of Cologne): Class numbers and representations by quadratic forms

In my talk, which reports on joint work with Ben Kane, I investigate various new relations between representations of integeger by quadratic forms (with congruence conditions). This is in the spirit of work of Petersson. There is an application to 7-core partitions.

Brumley, Farrell (Sorbonne University): Quantum ergodicity in the Benjamini–Schramm aspect for compact quotients of SL_n

On a compact Riemanian manifold M of negative curvature, the geodesic flow displays many chaotic features. In particular, the geodesic flow is ergodic, which implies the equidistribution of almost every orbit. The Quantum Ergodicity theorem of Schnirelman establishes a spectral version of this phenomenon: the

L^2 mass of almost every Laplacian eigenfunction equidistributes, in an appropriate sense, to the uniform measure on M.

In another direction, we can replace the variety M by a discrete version, such as a finite regular graph. As the spectrum of the latter is finite, we now consider the mass distribution of eigenfunctions in a fixed spectral interval as the graph varies in a family. Under an expansion hypothesis playing the role of negative curvature, Anantharaman and Le Masson proved quantum ergodicity for sequences of finite $(q+1)$-regular graphs whose geometry tends asympytotically towards their universal cover.

These two contexts can be interpreted uniformly, by taking a sequence of compact metric spaces M_n which converge, in the Benjamini—Schramm sense, to a limit space X. In this talk we shall be interested in the case where $M_n = \Gamma_n \backslash X$ is a sequence of locally symmetric spaces of non-compact type. Under an expansion condition, we again expect M_n to be quantum ergodic. In rank 1, this is a result of Le Masson—Sahlsten and Abert—Bergeron—Le Masson. In a work in progress with Jasmin Matz, we prove this conjecture in higher rank, when $X = \mathrm{SL}(n)/\mathrm{SO}(n)$.

Buttcane, Jack (University of Maine): Bessel functions outside GL(2)

I will discuss the definition and construction of the Bessel-type functions occurring in the Kuznetsov trace formula in a general setting, what is currently known about them, and their applications.

Diaconu, Adrian (University of Minnesota): Secondary terms in the asymptotics of moments of L-series

I will discuss a general conjectural asymptotic formula for moments of quadratic Dirichlet L-series. To illustrate the method, I will focus, for simplicity, on the analogous problem in the rational function field setting. This is joint work with Henry Twiss.

Finis, Tobias (University of Leipzig): Weyl's law with remainder term and Hecke operators

We are considering Weyl's law (the asymptotics of the number of Laplace eigenvalues) for the cuspidal spectrum of the locally symmetric spaces associated to congruence subgroups of arithmetic lattices, and in particular the magnitude of the remainder term. An interesting generalization is the asymptotics of Hecke operators, where one is asking for a quantitative estimate for the remainder term in terms of both the Laplace eigenvalue and the Hecke operator. Lindenstrauss-Venkatesh in 2007 first proved Weyl's law for the cuspidal spectrum of congruence subgroups of general adjoint Chevalley groups over the rationals (without a remainder estimate). Modifying their approach, and combining it with new estimates for the geometric side of Arthur's trace formula, we obtain a power saving in the remainder term for all adelic quotients associated to simple Chevalley groups over the rational number field, and a corresponding statement for the asymptotics of Hecke operators. This is joint work with Jasmin Matz (regarding the geometric estimates) and Erez Lapid.

Frączyk, Mikolaj (Institute for Advanced Study): Density conjecture for horizontal families of lattices

Let G be a real semi simple Lie group with an irreducible unitary representation π. The non-temperedness of π is measured by the parameter $p(\pi)$ which is defined as the infimum of $p \geq 2$ such that π has matrix coefficients in $L^p(G)$. Sarnak and Xue conjectured that for any arithmetic lattice $\Gamma \subset G$ and principal congruence subgroup $\Gamma(q) \subset \Gamma$, the multiplicity of π in $L^2(G/\Gamma(q))$ is at most $O(V(q)^{2/p(\pi)+\epsilon})$ where $V(q)$ is the covolume of $G(q)$. In a joint project with Gergely Harcos, Peter Maga and Djordje Milicevic we prove that such estimate holds uniformly for quite general natural families of non-commensurable lattices in $G = \mathrm{SL}(2, R)^a \times \mathrm{SL}(2, C)^b$. Our methods also apply to families of varying representations representations of G. For example, when the lattice is fixed, one of the components of π is fixed non-tempered and the remaining components vary among principal series representations we obtain power saving over the trivial bound on multiplicity (i.e. Weyl's law), where the quality of the saving improves with $p(\pi)$.

Goldfeld, Dorian (Columbia University): Orthogonality relations for coefficients of automorphic L-functions

We consider the average of the n^{th} Dirichlet coefficient (for arbitrary $n \geq 1$) in the family of L-functions attached to cusp forms on $\mathrm{GL}(r)$ for $r = 1, 2, 3, 4, \ldots$. This talk will review the history of this problem and then focus on my recent joint work with Eric Stade and Michael Woodbury where, for the first time, we prove an asymptotic formula with power savings in the case of $\mathrm{GL}(4)$.

Humphries, Peter (University College London): Small scale equidistribution of lattice points on the sphere

Consider the projection onto the unit sphere in $\mathbb{R}^3$ of the set of lattice points $(x_1, x_2, x_3) \in \mathbb{Z}^3$ lying on the sphere of radius $\sqrt{n}$. Duke and Schulze-Pillot showed in 1990 that these points equidistribute on the sphere as $n \to \infty$. We study a small scale refinement of this theorem, where one asks whether these points equidistribute in subsets of the sphere whose surface area shrinks as n grows. A particular case of this is a conjecture of Linnik, which states that for all $\delta > 0$, the equation $x_1^2 + x_2^2 + x_3^2 = n$ has a solution with $|x_3| < n^\delta$ for all sufficiently large n. We make nontrivial progress towards this, as well as proving an averaged form of this conjecture. This is joint work with Maksym Radziwiłł.

Kedlaya, Kiran (University of California, San Diego): An overview of the p-adic local Langlands correspondence

The global Langlands correspondence is supposed to relate Galois representations with automorphic representations, in a manner that admits a local-to-global compatibility. The latter appears when one starts with a representation of the Galois group of a number field, matches it with an adelic automorphic representation, and then restricts on the Galois side to a local field and on the automorphic side to one of the adelic factors. In the original setup, these two local fields are supposed to have different residue characteristics (corresponding to taking ℓ-adic etale cohomology of a scheme over a p-adic field, where $\ell \neq p$); we consider what is supposed to happen when these two characteristics coincide. In this case, we generally expect a very rich relationship between the two sides, but essentially only the case of $\mathrm{GL}_2(\mathbb{Q})$ is understood (by work of Colmez).

Khan, Rizwanur (University of Mississippi): Non-vanishing of Dirichlet L-functions

I will discuss recent progress towards the best known proportion for the non-vanishing of primitive Dirichlet L-functions of large modulus q, at the central point $s = 1/2$.

Lapid, Erez (Weizmann Institute): On Bernstein's proof of the meromorphic continuation of Eisenstein series

Lau, Yuk-Kam (University of Hong Kong): On the first negative Hecke eigenvalue of automorphic forms on $GL(2, \mathbb{R})$

Around a decade ago, Matomaki showed that the size of the first negative Hecke eigenvalue of a holomorphic primitive form of weight k and level N is bounded by $O((k^2N)^{3/8})$. We review this problem and the method of attack. Then we explore the application of this method in the case of primitive maass forms.

Michel, Philippe (École Polytechnique Fédérale de Lausanne): Some extensions of Duke's theorems

What is commonly referred as « Duke's theorems » are a collection of equidistribution results for representations of integers by ternary quadratic forms of various signatures on the associated quadratic surfaces. By duality these are equivalent to the equidistribution of certain torus orbits in locally homogeneous spaces attached to various quaternions algebras. In this talk, I will review Duke's theorems, the various approaches available (automorphic forms and/or homogeneous dynamics); I will also discuss a series of recent extensions (one joint with Menny Aka, Manuel Luethi and Andreas Wieser) which establish the simultaneous equidistribution of these torus orbits on products of these locally homogeneous spaces and use crucially the classification of joinings for certain diagonalizable actions due to Manfred Einsiedler and Elon Lindenstrauss.

Milićević, Djordje (Bryn Mawr College): Extreme values of twisted L-functions

Distribution of values of L-functions on the critical line, or more generally central values in families of L-functions, has striking arithmetic implications. One aspect of this problem are upper bounds and the rate of extremal growth. The Lindelöf Hypothesis states that $\zeta(\frac{1}{2}+it) \ll_\epsilon (1+|t|)^\epsilon$ for every $\epsilon > 0$; however neither this statement nor the celebrated Riemann Hypothesis (which implies it) by themselves do not provide even a conjecture for the precise extremal sub-power rate of growth. Soundararajan's method of resonators and its recent improvement due to Bondarenko–Seip are flexible first moment methods that unconditionally show that $\zeta(\frac{1}{2} + it)$, or central values of other degree one L-functions, achieve very large values.

In this talk, we address large central values $L(\frac{1}{2}, f \otimes \chi)$ of a fixed GL(2) L-function twisted by Dirichlet characters χ to a large prime modulus q. We show that many of these twisted L-functions achieve very high central values, not only in modulus but in arbitrary angular sectors modulo $\pi\mathbb{Z}$, and that in fact given any two modular forms f and g, the product $L(\frac{1}{2}, f \otimes \chi)L(\frac{1}{2}, g \otimes \chi)$ achieves very high values. To obtain these results, we develop a flexible, ready-to-use variant of Soundararajan's method that uses only a limited amount of information about the arithmetic coefficients in the family. In turn, these conditions involve small moments of various combinations of Hecke eigenvalues over primes, for which we develop the corresponding Prime Number Theorems using functorial lifts of GL(2) forms.

Munshi, Ritabrata (Tata Institute): New directions in the subconvexity problem

We will discuss the subconvexity problem for the Rankin-Selberg convolution of a GL(3) and a GL(2) form. I will demonstrate an uniform treatment which settles the problem in the t-aspect, the spectral aspect (in the generic case) and the twist aspect.

Petrow, Ian (University College London): The Weyl bound for Dirichlet L-functions

I will discuss work with Matt Young on the Weyl subconvex bound for Dirichlet L-functions. We prove this result by establishing a Lindelöf on average upper bound for a cubic moment of GL(2) L-functions along the lines of work of Conrey and Iwaniec. I will sketch the proof of the result assuming that the character chi has cube-free conductor. The extension of the result to all Dirichlet characters will be discussed by Matt Young in his talk, but the two presentations will be self-contained.

Popa, Alex (Simion Stoilow Institute of Mathematics): Multiple Dirichlet series for affine Weyl groups

Multiple Dirichlet series (MDS) have emerged as an important tool in number theory in the past 20 years. I will briefly explain their significance in the problem of giving asymptotic formulas for moments of central values of quadratic Dirichlet L-functions.

I will then describe an MDS introduced by Chinta and Gunnels in the function field setting, which has a group of functional equations isomorphic to the Weyl group of a reduced root system, finite or infinite. The simplest infinite case is that of an affine root system, and for a few such affine Weyl groups we find that the MDS satisfies a surprising additional symmetry besides the functional equations. As an application, one can use this symmetry to compute residues of the MDS in terms of infinite products, obtaining formulas which can be viewed as deformations of Macdonald's identity for the affine Weyl group. This is joint work with Adrian Diaconu and Vicenţiu Paşol.

Raulf, Nicole (University of Lille): Asymptotics of class numbers

In this talk we present results on the asymptotic behaviour of class numbers in the situation that the class numbers are ordered by the size of the regulator. We also will discuss the methods that can be used to obtain these results.

Risager, Morten (University of Copenhagen): Shifted convolution sums and small-scale mass equidistribution at infinity of Hecke eigenforms

We consider bounds on shifted convolution sums related to Hecke eigenforms. We discuss pointwise bounds and mean-square averages over different parameters. We then discuss how some of these mean-square averages have applications to small-scale mass equidistribution of Hecke eigenforms on sets shrinking towards infinity. This is joint work in progress with A. Nordentoft and Y. Petridis.

Saha, Abhishek (Queen Mary University): The Manin constant and p-adic bounds on denominators of the Fourier coefficients of newforms at cusps

The Manin constant c of an elliptic curve E over $\mathbb{Q}$ is the nonzero integer that scales the differential ω_f determined by the normalized newform f associated to E into the pullback of a Néron differential under a minimal modular parametrization $\phi\colon X_0(N)_{\mathbb{Q}} \twoheadrightarrow E$. Manin conjectured that $c = \pm 1$ for optimal parametrizations. I will talk about recent work that makes progress towards this conjecture by establishing an integrality property of ω_f necessary for this conjecture to hold. Our result implies in particular that $c \mid \deg(\phi)$ under a minor assumption at 2 and 3 that is not needed for cube-free N or for parametrizations by $X_1(N)_{\mathbb{Q}}$.

We reduce the above results to p-adic bounds on denominators of the Fourier expansions of f at *all* the cusps of $X_0(N)_{\mathbb{C}}$. We succeed in proving stronger bounds in the more general setup of newforms of general weight and levels by approaching the problem representation-theoretically. These results follow from sharp lower bounds that we prove for the p-adic valuations of the values of the Whittaker newform of GL_2 over a nonarchimedean local field of characteristic 0, using techniques that were originally developed by me in the context of the analytic sup-norm problem. For local fields of odd residue charactertistic, this allows us to ultimately reduce to the classical facts about p-adic valuations of Gauss sums. To overcome obstacles at 2, we analyze nondihedral supercuspidal representations of $\mathrm{GL}_2(\mathbb{Q}_2)$.

This is joint work with Kęstutis Česnavičius and Michael Neururer.

Sawin, Will (Columbia University): The sup-norm problem for automorphic forms over function fields

Let $\mathbb{F}_q$ be a finite field and $\mathbb{F}_q[T]$ the ring of polynomials in one variable over $\mathbb{F}_q$. There is a theory of modular forms invariant under congruence subgroups of $\mathrm{GL}_2(\mathbb{F}_q[T])$ that is analogous to the classical theory of modular forms invariant under congruence subgroups of $\mathrm{GL}_2(\mathbb{Z})$. We study an analogue of the classical sup-norm problem, which asks for bounds on the largest value of a cusp form, in this setting. We obtain, for forms of squarefree level with trivial central character, a bound stronger than the analogous bounds in the classical setting, as long as $q > 134$. This uses a new geometric method which should also apply to automorphic forms on more general groups over function fields. I will explain the background material and some of the key ideas that go into the proof.

Thorner, Jesse (University of Florida): Zeros of Rankin-Selberg L-functions and strong multiplicity one

Let F be a number field, and let $\mathfrak{F}_n$ be the set of all cuspidal automorphic representations of GL_n over F with (normalized) unitary central character. Let $n, n' \geq 1$ be integers, and let $\pi' \in \mathfrak{F}_{n'}$. I will discuss a new zero density estimate for the family of Rankin-Selberg L-functions $\{L(s, \pi \times \pi')\colon \pi \in \mathfrak{F}_n\}$. This shows that almost all Rankin-Selberg L-functions have a strong zero-free region near the line $\mathrm{Re}(s) = 1$. This work is unconditional; in particular, reliance on the generalized Ramanujan conjecture is avoided. I will emphasize an application to a strong average form of analytic multiplicity one on GL_n which extends the work of Duke and Kowalski beyond GL_2. I will briefly mention some applications regarding hybrid-aspect subconvexity for the central values of almost all Rankin-Selberg L-functions.

Young, Matt (Texas A&M University): The fourth moment of Dirichlet L-functions along a coset

I will discuss recent work with Ian Petrow on the problem of bounding the fourth moment of Dirichlet L-functions on a coset of a subgroup of characters modulo d inside the full group of Dirichlet characters modulo q. The final result is analogous to Iwaniec's bound on the fourth moment of the Riemann zeta function along a short interval. This fourth moment bound in turn has an application to the Weyl bound for all Dirichlet L-functions via a cubic moment (discussed by Ian Petrow in his talk), but this presentation will be self-contained.