Title: On large κ-homogeneous, but not κ-transitive permutation groups

A permutation group G on A is κ-homogeneous iff for all $X, Y \in [A]^\kappa$ with $|A \setminus X| = |A \setminus Y| = |A|$ there is a $g \in G$ with $g[X] = Y$. G is κ-transitive iff for any injective function f with $dom(f) \cup ran(f) \in [A]^{\le\kappa}$ and $|A \setminus dom(f)| = |A \setminus ran(f)| = |A|$ there is a $g \in G$ with $f \subset g$.

The starting point of our investigation is two classical questions of P. M. Neumann:

(1) Is it true that a κ-homogeneous permutation group should be κ-transitive?

(2) Is it true that if a permutation group is κ-homogeneous, then is also λ-homogeneous for all $\lambda < \kappa$?

After surveying some classical results we give a partial answer to one of the questions. we show that there is an ω-homogeneous but not ω-transitive permutation group on a cardinal λ provided

(a) $\lambda < \omega_\omega$, or

(b) $2^\omega < \lambda$, and $\mu^\omega = \mu^+$ and $\Box_\mu$ hold for each $\mu \le \lambda$ with $\omega = cf(\mu) < \mu$, or

(c) our model was obtained by adding ω_1 many Cohen generic reals to some ground model.

For $\kappa > \omega$ we give a method to construct large κ-homogeneous, but not κ-transitive permutation groups. Using this method we show that there exists κ^+-homogeneous, but not κ^+-transitive permutation groups on κ^{+n} for each infinite cardinal κ and natural number $n \ge 1$ provided $V = L$.