The profinite topology of free groups and weakly generic tuples of automorphisms

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In the last decades a considerable effort has been made in order to study infinite relational structures that can be constructed from their finite substructures. The aim of the present work is to survey some recently obtained related results that might provide a better understanding of this problem in some special cases (when the infinite structure, in some sense, is symmetric enough).

More concretely, let $\mathcal{A}$ be a countable first order structure and endow the universe of $\mathcal{A}$ with the discrete topology. Then the automorphism group $Aut(\mathcal{A})$ of $\mathcal{A}$ becomes a topological group. A tuple of automorphisms $\langle g_0, ..., g_{n-1}\rangle$ is defined to be weakly generic iff its diagonal conjugacy class (in the algebraic sense) is dense (in the topological sense) and further, the $\langle g_0, ..., g_{n-1}\rangle$-orbit of each $a \in A$ is finite. Existence of tuples of weakly generic automorphisms are interesting from the point of view of model theory as well as from the point of view of finite combinatorics.

The main results of the present work are as follows. We characterize the existence of tuples of weakly generic automorphisms in terms of the profinite topology of the free groups. We will show that if $Aut(\mathcal{A})$ has finite topological rank r (and satisfies a further, mild technical condition) then the existence of a weakly generic tuple in $Aut(\mathcal{A})^r$ implies the existence of weakly generic tuples in $Aut(\mathcal{A})^n$ for all natural number $n \geq 1$. Finally, we show, that if $\mathcal{A}$ is a countable model of an $\aleph_0$-categorical, simple theory in which all types over the empty set are stationary and $\mathcal{A}$ has a pair of weakly generic automorphisms, then it has tuples of weakly generic automorphisms of arbitrary finite length.