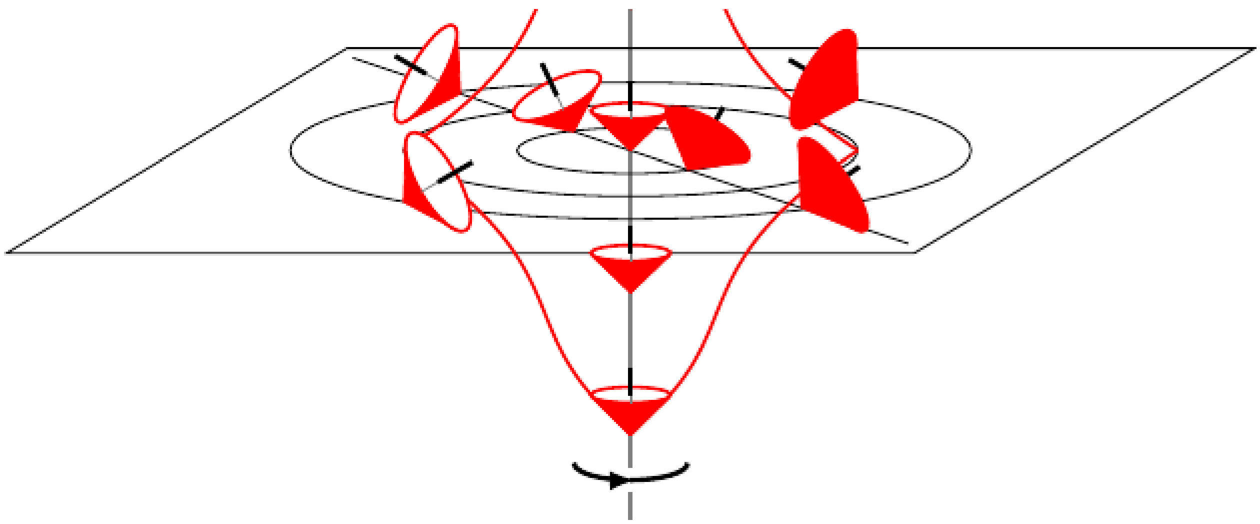
Logic, Relativity and Beyond
4th International Conference

Online 14 July – 17 July 2021

$$\mathfrak{M}_{\text{Gödel}} \models \mathit{GenRel}$$

$$\mathit{GenRel} \not\models \neg\exists \mathit{CTC}$$



Volume of Abstracts

Alfréd Rényi Institute of Mathematics 2020 - 2021

Logic, Relativity and Beyond
4th International Conference

Online 14 July – 17 July 2021

There are several new and rapidly evolving research areas blossoming out from the interaction of logic and relativity theory. The aim of this conference series, which take place once every 2 or 3 years, is to attract and bring together mathematicians, physicists, philosophers of science, and logicians from all over the world interested in these and related areas to exchange new ideas, problems and results. The spirit of this conference series goes back to the Vienna Circle and Tarski's initiative Logic, Methodology and Philosophy of Science. We aim to provide a friendly atmosphere that enables fruitful interdisciplinary cooperation leading to joint research and publications.

Topics include (but are not restricted to):

- Special and general relativity
- Axiomatizing physical theories
- Foundations of spacetime
- Computability and physics
- Relativistic computation
- Cosmology
- Relativity theory and philosophy of science
- Knowledge acquisition in science
- Temporal and spatial logic
- Branching spacetime
- Equivalence, reduction and emergence of theories
- Cylindric and relation algebras
- Definability theory
- Concept algebras and algebraic logic

https://conferences.renyi.hu/lrb20/

The 4th Logic, Relativity and Beyond International Conference was supposed to be held at Fried Castle Resort (Simontornya, Hungary). Due to the pandemic of Covid-19, the conference was delayed and then it was decided to carry on the conference online. Link to the online event: https://conferences.renyi.hu/lrb20-online.

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This conference is organized by Alfréd Rényi Institute of Mathematics, Budapest.

Schedule and Program

The schedule below is in Central European Time (CET). An online calendar version of the program automatically fitting to your local time is available (and downloadable in .ics format) from:
https://framadrive.org/apps/calendar/p/SgNfJgyG7Ns5Qxxr/listMonth/2021-07-14

Wednesday, 14 July 2021

Session 1 (Chair: Gergely Szekely)

16:20 - 16:30	**Opening**
15:30 - 16:00	**When generalised definitional equivalence implies definitional equivalence** *Laurenz Hudetz*
17:00 - 17:30	**Axiomatic and Genetic Methods of Concept- and Theory-Building: An Attempt of Synthesis** *Andrei Rodin*
17:30 - 18:00	Break

Session 2 (Chair: Mike Stannett)

18:00 - 18:30	**Rotating black holes as time machines: a re-assessment** *Juliusz Doboszewski*
18:30 - 19:00	**Where Does General Relativity Break Down?** *James Weatherall*
19:00 - 19:30	**On Surplus Structure Arguments** *Samuel Fletcher*

Thursday, 15 July 2021

Session 1 (Chair: Koen Lefever)

16:30 - 17:00	**Concept Algebras and Conceptual Distance** *Mohamed Khaled*
17:00 - 17:30	**Concept algebra of special relativistic spacetime** *Judit Madarász*
17:30 - 18:00	Break

Session 2 (Chair: Mohamed Khaled)

18:00 - 18:30	**Comparing classical and relativistic dynamics in terms of inelastical collisions** *Koen Lefever and Gergely Székely*
18:30 - 19:00	**Now, can or cannot classical kinematics interpret special relativity?** *Gergely Székely*
19:00 - 19:30	**Predicate Logic with Explicit Substitution** *Richard Thompson*

Friday, 16 July 2021

Session 1 (Chair: Márton Gömöri)

15:00 - 15:30	**Omitting types in finite variable fragments of first order logic** *Tarek Sayed Ahmed*
15:30 - 16:00	**A Solution to an Insoluble Problem** *Selmer Bringsjord, Naveen Sundar Govindarajulu and Atriya Sen*
16:00 - 16:30	Break

Session 2 (Chair: Mohamed Khaled)

16:30 - 17:00	**On the gap between definitional and categorical equivalence of theories** *Hajnal Andréka, Judit Madarász, István Németi, Péter Németi and Gergely Székely*
17:00 - 17:30	**Machine Verification of the No-FTL-Observer Theorem for First-Order General Relativity** *Hajnal Andréka, Edward Higgins, Judit X. Madarász, István Németi, Mike Stannett and Gergely Székely*
17:30 - 18:00	Break

Session 3 (Chair: Koen Lefever)

18:00 - 18:30	**What exactly does the special principle of relativity state? A discussion of Einstein's 1905 paper** *Márton Gömöri*
18:30 - 19:00	**Why did such serious people take so seriously axioms which now seem so arbitrary?** *László E. Szabó, Márton Gömöri and Zalán Gyenis*
19:00 - 19:30	**Several Steps in the Procedure of Hilbert's Axiomatic Method** *Giambattista Formica*

Saturday, 17 July 2021	
Session 1 (Chair: Mike Stannett)	
15:00 - 15:30	**Lorentzian Structures on Branching Spacetimes** *David O'Connell*
15:30 - 16:00	**The weak correspondence principle: a new intertheory relation in physics based on Rosaler's empirical reduction** *Marcoen Cabbolet*
16:00 - 16:30	Break

Session 2 (Chair: Gergely Székely)

16:30 - 17:00	**Prospects for Possibilism** *Sebastián Gil*
17:00 - 17:30	**Do you see what I see? Joint observation in Barbourian universe** *Petr Švarný*

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Contributed Talks

Machine Verification of the No-FTL-Observer Theorem for First-Order General Relativity

H. Andréka, E. Higgins, J.X. Madarász, I. Németi, M. Stannett, and G. Székely

December 2019

We have previously presented a machine-verified proof of the No-FTL-Observer theorem for first-order special relativity theory, SpecRel, which states that no observer ever observes another observer to be moving faster than the speed of light [8–10]. This was constructed in the context of a growing body of work on axiomatic computational formalisations of physics and their use in discovering, verifying and analysing physical theorems [1–7].

The main goal of this new study is to extend our earlier verification to include the corresponding situation in GenRel, the first-order formalisation of the general theory. Although our earlier proofs were based in the formal axioms of SpecRel, we relied on a large number of low-level lemmas describing basic geometrical results. For the new proofs we are endeavouring to retain the high-level structure of paper-based GenRel proofs, where intuition is mitigated by extended derivations from the axioms alone, with as little low-level knowledge intruding as possible. We consider this an essential feature moving forward to the development of the new proofs. To see why, consider the following lemma:

> *Suppose $\vec{p}$ and $\vec{v}$ are vectors in Q^4 , where Q is the field of quantities. If ℓ is the line comprising all points with position vectors in the set $\{\vec{p} + \lambda\vec{v} \colon \lambda \in Q\}$ then ℓ passes through the point whose position vector is $\vec{p}$.*

This low-level statement is both obvious and easy to prove, and results of this kind were frequently used in our earlier SpecRel proofs. They nonetheless involve the interactive construction of formal proofs, and this can be very time consuming – especially for newcomers to interactive proof generation. Moreover, their use was largely motivated by existing intuitions as to how geometrical structures 'ought to work' in Euclidean and Minkowskian geometry. This reliance on physical intuition to guide proofs is clearly more problematic as we move to the more complex situations found in general relativistic settings where intuition frequently proves unreliable. Ultimately, therefore, we want to place more emphasis on extracting as much information as possible from the observer-axioms of GenRel, with less reliance on the low-level algebraic properties of the field Q.

We will first outline the first-order axioms underpinning the proof, and justify why they are appropriate. We then present the proof informally, before discussing the progress made so far in constructing the machine-verified version. As in our earlier work on the special theory the machine proof is intended to follow the formal paper-based version closely, but we also expect to need various auxiliary lemmas whose main role is to simplify the proof process rather than provide physical insight. We will look at these in detail, and consider the extent to which our goal of retaining a high-level perspective can be achieved.

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[1] Naveen Sundar G., Selmer Bringsjord, and Joshua Taylor. Proof verification and proof discovery for relativity. In *First International Conference on Logic and Relativity: honoring István Németi's 70th birthday.* Rényi Mathematical Institute, June 2012. `http://www.renyi.hu/conferences/nemeti70/Govindarajulu-Bringsjord.pdf`, see also [2].

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On the gap between definitional and categorical equivalence of theories

Andréka, H., Madarász, J., Németi, I., Németi, P. and Székely, G.

In [2], the relationships between three notions of sameness of first-order theories is investigated. These three notions are definitional equivalence, Morita equivalence, and categorical equivalence of theories. It is proved in [2] that they are strictly weaker in the listed order. We note that [5] proposes an interesting different kind of continua of notions between the two extremes of this list, this continua of notions goes also beyond first-order languages. The notion of categorical equivalence is further discussed in [8].

We present two results in connection with [2]'s three-member hierarchy. We use the terminology of [2].

Theorem 1 *There are two first-order logic theories T_1 and T_2 on finite vocabularies that are categorically equivalent but not Morita equivalent. Moreover, (i)-(ii) below hold.*

(i) There is a functor F between the model categories of T_1 and T_2 that

- *is an isomorphism,*
- *commutes with the natural forgetful functors to* Set*, and*
- *preserves ultraproducts up to isomorphisms.*

(ii) T_1 and T_2 are not just not Morita equivalent, they are not even bi-interpretable in the sense of [4].

Theorem 1 answers [2, Question 6.1].

Our second theorem shows that if a functor F as in Theorem 1 preserves ultraproducts exactly and not only up to isomorphisms, then it makes the two theories T_1 and T_2 definitionally equivalent.

Theorem 2 *Assume that F is a functor between the model categories of T_1 and T_2. Then (i) and (ii) below are equivalent.*

(i) The functor F

- *is an isomorphism,*
- *commutes with the natural forgetful functors to* Set*, and*
- *preserves ultraproducts.*

(ii) T_1 and T_2 are definitionally equivalent.

Theorem 2 above gives an answer to the conjecture formulated below Corollary 2 in [1]. Theorem 2 is a generalization of the first theorem in [3], and it is analogous to Makkai's famous theorem about ultracategories [7, Theorem 4.1].

It would be most interesting to find analogous theorems concerning the gap between categorical and Morita equivalence, and even more interesting, to give an analogous theorem that bridges the gap between Morita equivalence and definitional equivalence. In this connection, see [6].

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Special Relativity and Theoretical Equivalence

Quine [1975] has proposed an attractive criterion[1] for when two first-order systems count as axiomatizations of the same [scientific] theory. He asked for a translation function - meeting certain logical criteria - between the languages of the two theories. One set of axioms is to be mapped into a logical equivalent of the other. Metasemantical considerations - considerations about the way in which the reference of theoretical terms gets fixed - suggest that Quine's [1975] formal notion of equivalence entails strong forms of ontological equivalence.[2] No rational preference can be had for one equivalent theory over the other, and the ontology of the two theories is the same. Unfortunately, few of the cases that one would like to look at - such as Lagrangian and Hamiltonian mechanics, or matrix and wave quantum mechanics, or the manifold and the algebraic formulation of general relativity - are amenable to this kind of detailed analysis. The main obstacle is a lack of axiomatizations. In the present work, we attempt to make progress on two fronts. (1) We propose an improvement on Quine's [1975] original notion of equivalence. We admit mappings that reconstrue predicates of given arity into predicates of larger arity.[3] (2) We study two systems of axioms for relativity with apparent different ontological import. One is a revision of the system of [Andréka, Madarász et al. 2011. The second is an attempt to formalize the geometric account of spacetime in [Maudlin 2012] and [Malament, unpublished]. We will present the axioms of both system and (3) proceed to prove their equivalence by constructing an appropriate translation manual.

Our adaption of the system of [Andréka, Madarász et al., 2011] is framed in terms of a six place predicate W(o, b, x, y, z, t) for 'observer o assigns to body b the coordinates x, y, z and t'. One can eliminate their other predicates by defining an observer as whatever coordinatizes, a body as whatever is coordinatized, and a number as whatever is assigned by an observer to a body. A first set of axioms for numbers are those for a real closed field. A second group deals with the assignment of coordinates to photons and bodies relative to inertial observers. The main axiom asserts that every two frames are related by sixteen numbers defining a Lorentz transformation. For every frame and for every choice of sixteen numbers (defining a Lorentz matrix), there is a second frame related to the first by the Lorentz matrix. The second *geometric* theory is formulated with primitive predicates for betweenness and to compare the relativistic interval between points. $\mathrm{Bet}(x, y, z)$ means 'y is between x and z on a line', and $<_{\equiv} (x, y, z, w)$ means 'the square of the interval between x and y is shorter than between z and w'. In formulating the axioms, we have found it easier to deal with the square of the interval between points rather than with the interval. The system does not constitute a strictly more powerful system for Minkowski spacetime than those of [Pambuccian 2007] and [Goldblatt 1987], but it is easier to see how to develop deductively a good deal of physics in it.

A first set of axioms for the notion of betweenness - which characterizes four dimensional affine spaces - are imported from [Szczerba and Tarski 1979]. The other axioms for the relativistic interval require a battery of preliminary definitions. Two points are lightlike connected

[1] Later refined by Barrett and Halvorson [2016]. Barrett and Halvorson [2016] ask also for a reverse function. The composition of the reconstrual and this reverse reconstrual must always return a logically equivalent formula.

[2] We have in mind the view that theoretical terms are implicitly defined and in particular the ramseyfication approach of [Lewis 1970] - amended to take into account a combination of *naturalness* and *charity*.

[3] This extension can be motivated by examples from mathematics: the interpretation of the theories of matrices and of polynomials in their [field of] coefficients, that of the rationals in that of the integers and the equivalence of geometrical theories about lines and points. [Halvorson 2019, pp.143-146]

when the interval between them is null: congruent to the segment between a point and itself. Similarly, we can define timelike and spacelike connection, according to whether the interval is shorter or longer than a null one. Orthogonality is the cornerstone of our system. It is needed to define the notions of linear algebra. We define it by cases. When $\overline{xy}$ is spacelike and $\overline{xz}$ is timelike, for example, the two are orthogonal just in case the 'longest' route from a point w collinear to $\overline{xy}$ to $\overline{xz}$ passes through x. Similar conditions apply in other cases. A second key notion is the relation $Opp(x, y, z, w)$ obtaining between a timelike and spacelike vector when the square of the interval - or what we improperly call the 'length' of these segments - differ only in term of 'sign'. They are of equal absolute value but of different type. To check whether this relation obtains, we need to transport one of the segments onto a congruent one that is orthogonal to the first. Our first two segments are of opposite length if the sum of the displaced ones is a null vector i.e the diagonal of the square has length zero. Let now $\mathbf{E_3}$ be Tarski's system for three-dimensional euclidean geometry, and let $\mathbf{E_3} \setminus \{\text{As.11}\}$ be [the conjunction of] its axioms except for the the schema of continuity. Our main axiom states that at every point there are four others such that: (1) three are spacelike and one is timelike connected to the first, (2) these segments span the entire space (in the sense familiar from linear algebra), (3) the hyperplane spanned by the spacelike segments obeys the axioms of Euclidean geometry i.e we assert $\mathbf{E_3} \setminus \{\text{As.11}\}$ with quantifiers restricted to the plane. *Summation axioms* specify the relativistic length of arbitrary segments as a function of their decomposition on a basis. We postulate that a segment extending a spacelike segments is spacelike and shorter. A segment extending a timelike segment is timelike and longer. *Segment construction* axioms guarantee the existence of segments of arbitrary lengths. The last piece are *axioms of continuity*.

To translate the theory of [Andréka, Madaràsz et al., 2011] we define addition and multiplication by standard methods [Schwabhäuser, Szmielew and Tarski and 1983, pg. 160]. Statements about an observer will be paraphrased as statements about a quintuple of points [with five free variables]: an origin and four points on orthogonal axes. The definition of W(o, b, x, y, z, t) is an adaptation of the construction of coordinates in [Maudlin 2012].

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A Solution to an Insoluble Problem

— extended abstract of `0103202215NY`**—**

Selmer Bringsjord • Naveen Sundar G. • Atriya Sen

In his "An Insoluble Problem," Storrs McCall (2010) claims to have derived from a Dummett-penned time-travel scenario of 1986, with help from an added dash of Gödel (1949), just that. Subsequently, McCall (2014) confidently offered a prize to anyone who can find a solution to the problem in question, which, in a word, is to find "who or what creates" the aesthetic masterpieces in the scenario. Then, in 2016, Bourne & Bourne published what they purported to be a solution to McCall's puzzle, and claimed their prize. This provoked a reply from McCall (2017) himself, in which he holds that they have in no way provided a solution, and hence he gave no prize. We agree with McCall that Bourne & Bourne have failed, but he is wrong that his problem is insoluble, because we have a solution. In the present abstract, we convey the gist of each of the six salient points in our analysis (the sixth point being the solution itself).

In order to informatively summarize herein each of these points, it's first necessary to convey Dummett's sci-fi scenario ($= s_1$), and to then make plain the puzzle based upon it that McCall declares to be insoluble.

Dummett (1986) described his scenario in an essay he wrote to make points orthogonal to those with which we are concerned herein, but it's important to note that in this essay Dummett certainly does assert that which McCall seeks to extract from it: viz., that it's rational to believe such scenarios as s_1 are physically possible. Here's s_1 (neatened and massaged a bit by us; quotes are from Dummett 1986):

> Twenty-five-year-old Art, a "fifth-rate but conceited" painter, is visited on a Monday in the year 1958 by a time-traveling art critic from the year 2058. The critic, Chris, sent to interview Art, explains that Art is regarded to be the greatest painter of the 21st century. When Art "proudly produces his paintings for inspection, the critic's face falls, and he says, in an embarrassed manner, that the artist cannot yet have struck the inspired vein in which he painted his (subsequently) celebrated masterpieces, and produces a portfolio of reproductions he has brought with him." Chris then must leave, and doesn't take the portfolio with him, which Art copies in paint for the rest of his career. The result, of course, is that he becomes celebrated as a seminal master of the art form.

We said above that McCall adds a Gödelian element to Dummett's story. Dummett's (1986) objective is merely to dissolve, at least in part, the stigma of absurdity associated with backwards causation,[1] and his *modus operandi* is philosophical analysis, expressed in English; no formal logic — in contrast with with our approach — is employed. To reach his objective, Dummett must establish that it's rational (or at minimum not irrational) to believe that such looping stories are physically possible. But McCall makes a much stronger claim: viz., that loops of the sort that Chris travels are *known* to be physically possible, in light of Gödel's (1949) having proved that certain solutions to Einstein's field equations entail the consistency of what are now known as "closed timelike curves" (CTCs).

Now, to finish the context needed to list the six prominent points in our coming full paper (and presentation in Hungary, if accepted), here is how the supposedly insoluble problem is posed, in McCall's own words:

[1] Which is why the one issue that consumes the most space in his essay is Newcomb's Problem (first introduced, as far as the lead author knows, in Nozick 1970), long understood to at least raise the "spectre" of backwards causation.

> Given the right circumstances, time travel is possible. There is no conceptual obstacle to understanding (i) the critic's visit to the artist, nor (ii) how the artist paints the copies. What is incomprehensible is (iii) who or what creates the works that future generations value? Where is the artistic creativity to be found? Unlike the traditional "paradoxes of time travel," this problem has no solution. (McCall 2010, p. 218)

With enough background now laid, here is the list of the salient points in our paper, in the order in which they unfold therein:

1. *Rectify the historical record.* We first explain that in point of fact, despite what many think, Dummet didn't originate the "looping artist" scenario, nor is it the case that the story's origin's are mysterious. Dummett's s_1 is (slightly adapted, presumably unwittingly) from a clever science-fiction story written by William Tenn (1955).

2. *Clarification of what, declaratively put, Gödel proved.* While there are geometric interpretations of Gödel's result (as Gödel himself points out in his original paper), the fact remains that the nature of this work is (shall we say) overtly and (for some) oppressively numerical. Fortunately, the landscape of the intersection of formal logic and theoretical physics has of late progressed in favor of a mode of analysis quite congenial to logicians and philosophers, including those among this group who are computationally inclined. We refer to the fact that physics has been substantively axiomatized (e.g. see Andréka et al. 2007). The second prominent point in our paper is to explain, by relatively simple deployment of modal logic, that, where $\mathscr{G}$ is a declarative expression of general relativity, Gödel proved that under the supposition of $\mathscr{G}$, CTCs are logically physically possible (not simply logically possible,[2] and not simply physically possible). Two alethic modal operators, the familiar $\Diamond$ for 'logically possibly,' plus $\blacklozenge$ for 'physically possibly,' are therefore needed. Let '$\mathcal{A}^\star$' denote *all* the particulars of our actual physical universe expressed declaratively. let 'CTCs:T' as refer to declarative content expressing that CTCs of the Gödelian type T happen. In addition, we abbreviate the deductive consistency of some set Φ of declarative statements by 'Con Φ.' We can then confidently say that any proof of

$$(+) \quad \text{Con } (\mathscr{G} \cup \mathcal{A}^\star \cup \text{CTCs:T})$$

would establish that

$$\blacklozenge \text{ CTCs:T.}$$

However, this consistency is not what Gödel established. Rather, where $\mathcal{A}$ is some proper subset of $\mathcal{A}^\star$ such that $\mathcal{A} \nvdash \mathcal{A}^\star$, he proved this:

$$(*) \quad \text{Con } (\mathscr{G} \cup \mathcal{A} \cup \text{CTCs:T}).$$

From this result, it can only be derived that:

$$\Diamond\blacklozenge \text{ CTCsT.}$$

We amend McCall's argument so that it's more accurate, because it clearly employs specifically this upshot from Gödel's result.

3. *Shift from copying painting to copying strings.* Bourne & Bourne (2016) suggest four options to meet McCall's challenge to find an explanation; one is to hold that, actually, copying a painting requires creativity. We eliminate this option by moving from painting to copying strings. Since it's far from clear that copying (say) a da Vinci painting obviates the need for any creativity (consider e.g. Figure 1 and its caption) , we take a cue from those in the formal sciences who have characterized algorithms as by definition devoid of creativity,[3] and accordingly supplant s_1 with

[2] We assume that "no conceptual obstacle" (McCall's phrase; see the quote above) to ϕ should be interpreted as 'it's logically possible that' ϕ.

[3] E.g. from the classic *Computability and Unsolvability*:

> [W]e shall be concerned with the problem of the existence of *algorithms* or *effective computational procedures* for solving various problems. What we have in mind are sets of instructions that provide

s_2, in which Art is a fifth-rate *novelist* whose output, by the time the eventful Monday arrives, has included nothing more than self-published, entirely formulaic fiction of the lowest order. The art critic brings with him three first-person novels with syntax as intoxicatingly intricate and precise as Proust, and with emotion, plot, personality, and theme as unforgettable as Proust's as well . This time, copying consists in executing an algorithm (f_c) for character-by-character copying; qua algorithm, the absence of creativity appears to be clearly secured. Figures 2 and 3 gives a pictorial depiction of the sequence that constitutes what we call "The Paradox of Proust." This is of course called a paradox because, following McCall, it seems that there is simply no explanation to be had for where literary creativity comes from in this new scenario.

4. *Discuss and refute the multiverse option for solving the problem.* Surprisingly, neither McCall nor the Bournes given any indication they are aware of the fact that some proponents of the multiverse interpretation of quantum mechanics have pinned their hopes on this interpretation for resolving (what we are now calling) 'The Paradox of Proust."[4]

5. *Proceed to destroy the remaining three options Bourne & Bourne (2016) claim each provide a solution to McCall's puzzle.* No further information provided in the present abstract, due to desired economy.

6. *Present and defend our solution to The Paradox of Proust'.* An entirely different line of thought to address the paradox has been ignored up til now. This line is launched by simply taking note of the fact that physics, however rich and wonderfully represented, formalized, and thus rendered suitable for rigorously reasoning about, by definition cannot cover that which is non-physical. Hence, if cognition, and in particular creativity, is non-physical, it's inevitable that there exist scenarios such that purely physical accounts of them fail to fully explain objects appearing in them. There is an analogy between this line of thinking, and (Bringsjord et al. 2001). No further information is provided in the present abstract, due to desired economy.

> mechanical procedures by which the answer to any one of a class of questions can be obtained. Such instructions are to be conceived of as requiring no "creative" thought in their execution. (Davis 1982/1958: xv)

For a concordant, longer, and wonderfully lucid characterization of the creativity-less nature of algorithms, see (McNaughton 1982).

[4]E.g., we read:

> In the art critic story quantum mechanics allows events, from the participants' perspective, to occur much as Dummett describes. The universe that the critic comes from must have been one in which the artist did, eventually, learn to paint well. In that universe, the pictures were produced by creative effort, and the reproductions were later taken to the past of another universe. There the paintings were indeed plagiarized — if one can be said to plagiarize the work of another version of oneself — and the painter did get "something for nothing." But there is no paradox, because now the existence of the pictures were caused by genuine creative effort, albeit in another universe. (Deutsch & Lockwood 1994 p. 74)

Figure 1: A Variant, by KB Foushée, of "Lady with an Ermine" by Leonardo da Vinci. *In this work, the artist of course intentionally diverges from the original in creative ways. But the point is that were the artist to endeavor to carefully copy the original da Vinci, working in paint, it's entirely possible that this effort would include irrrepressible creative moves. Or to put the point more circumspectly: We don't* ***know*** *that such copying doesn't include creativity. In stark contrast, algorithmically copying a string of characters can be counted upon to be bereft of any creativity, for sure.* (The artist's web site: https://kbfoushee.com.)

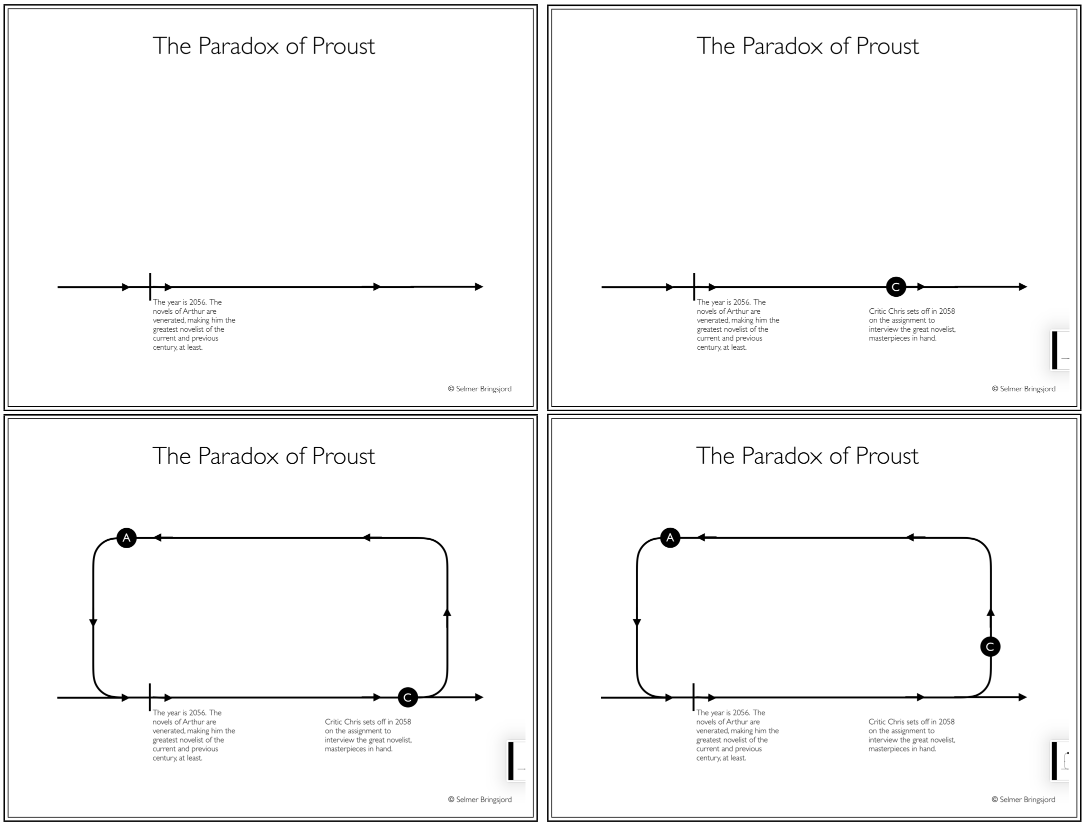

Figure 2: Sequence (Part I) in "The Paradox of Proust," Depicted Visually'

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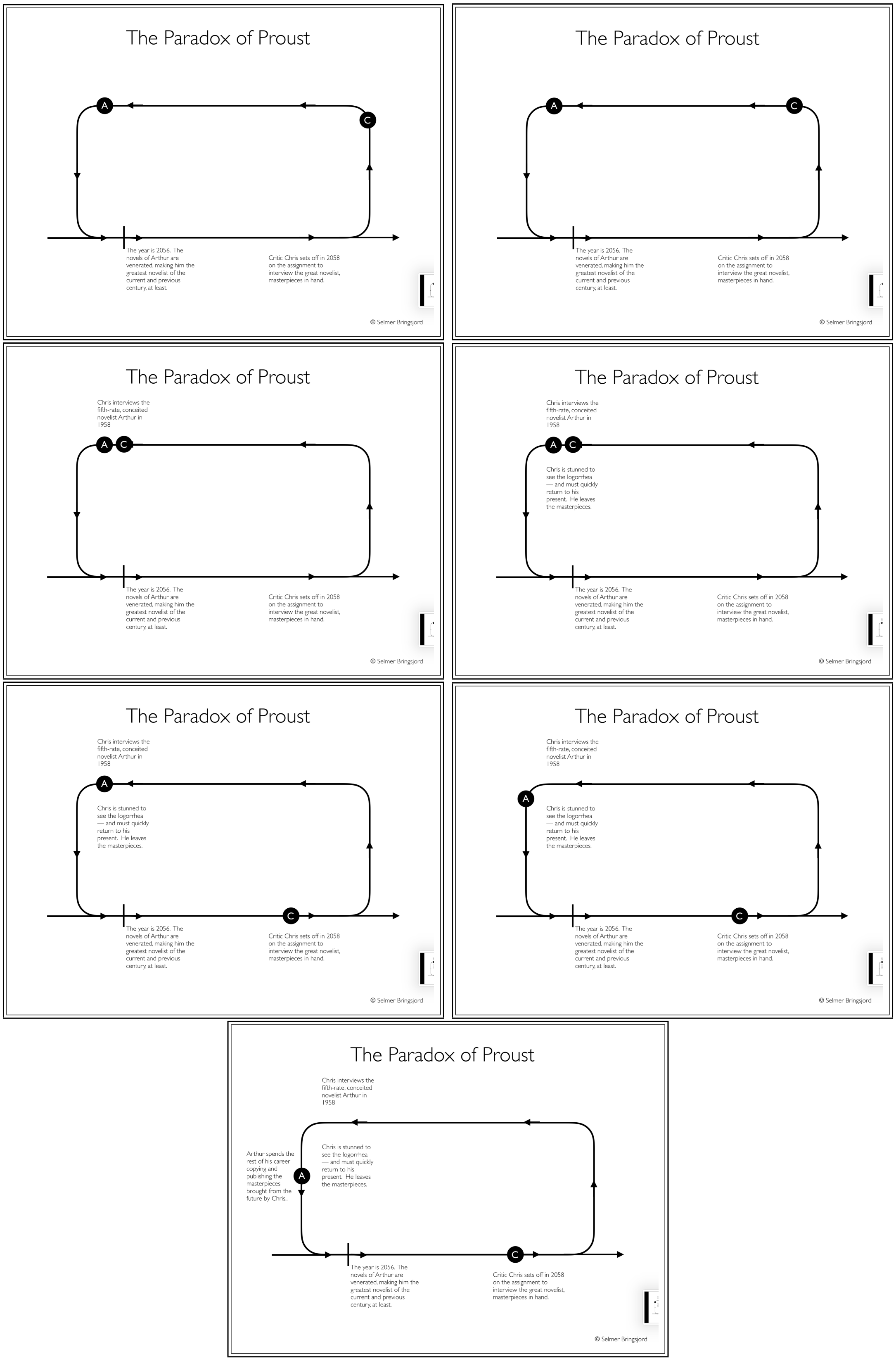

Figure 3: Sequence (Part II) in "The Paradox of Proust," Depicted Visually'

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The weak correspondence principle: a new intertheory relation in physics based on Rosaler's empirical reduction

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It is widely believed among physicists that any fundamentally new theory of physics **has to** satisfy the correspondence principle, meaning nothing else than that the new theory has to reduce to established theories of physics by applying some limit procedure. However, in this talk I will argue that there is another correspondence principle by which a new theory of physics can be in agreement with established theories of physics: I will call this the 'weak' correspondence principle in contrast to the aforementioned 'strong' correspondence principle.

In 2015 Rosaler introduced the general notion of 'empirical reduction', which is to be distinguished from the more familiar notion of 'formal reduction' [1]. What we have is that a new theory T *reduces formally* to an established theory T' if and only if T' is in some sense a special or a limit case of T—to prove a formal reduction, a mathematical analysis of T and T' is sufficient. Thus speaking, the strong correspondence principle can be expressed in terms of a formal reduction: a new theory T corresponds strongly to a theory T' if and only if T reduces formally to T'. By contrast, a new theory T *reduces empirically* to an established theory T' if and only if T reproduces the empirically successful predictions of T': outside the established area of application, T does not have to reproduce the predictions of T'. Obviously, formal reduction implies empirical reduction, but not vice versa'—the latter is thus a weaker intertheory relation. That being said, we can now express the weak correspondence principle in terms of empirical reduction: a theory T corresponds weakly to a theory T' if and only if T has a model M that reduces empirically to T'.

This weak correspondence principle plays a central role in research on the Elementary Process Theory (EPT): this is a collection of seven well-formed formulas written in a well-defined formalism, which are interpreted as generalized process-physical principles [2]. This gives an exact yet rather abstract view on the individual processes by which the smallest massive systems in nature evolve. Due to its degree of abstractness, the EPT cannot possibly correspond strongly to modern interaction theories. To show that the interactions as we know them from modern physics can nevertheless take place in the elementary processes described by the EPT, the weak correspondence principle comes into play: the idea is that an interaction described by theory T can take place in the elementary processes described by the EPT if and only if the EPT corresponds weakly to T. So, the first-order expressions

$$M \models A^i_{EPT} \tag{1}$$

$$M \models P^j_T \tag{2}$$

must then obtain for each of the seven axioms $A^1_{EPT}, \ldots, A^7_{EPT}$ of the EPT and for each of the n empirically successful predictions $P^1_T, \ldots, P^n_T$ of T expressed in the language of a model M. The EPT is then a *unifying scheme* if it has a model M that reduces empirically to both GR and QED. Thus speaking, the main aim of research on the EPT is to (dis-)prove that it is a unifying scheme.

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[1] J. Rosaler, *Topoi* **34**, 325-338 (2015)

[2] M.J.T.F. Cabbolet, *Ann. Phys. (Berlin)* **522**, 699-738 (2010); **523**, 990-994 (2011); **528**, 626-627 (2016)

Rotating black holes as time machines: a re-assessment

Maximally extended Kerr spacetime has regions containing closed timelike curves. Due to astrophysical importance of the exterior Kerr spacetime, it may be advisable to keep tabs on the viability of rotating black holes as candidates for a time machine spacetime; this, in turn, crucially depends on the presence of Cauchy horizons. In this context, my talk will (i) report on some of the relevant recent developments in mathematical physics, and (ii) offer a re-assessment of the prospects for viewing the Kerr spacetime as a time machine. (Most of these concerns will apply *salva veritate* to the issue of viability of Kerr spacetime as a candidate for relativistic hypercomputation or a form of Malament-Hogarth machine, see Manchak (2017), Andréka et al. (2018).) I will focus on three types of issues: "no hole" conditions, cosmic censorship hypothesis, and compatibility with thermodynamics and quantum field theory in curved spacetime. While the situation is far from definite, the overall prospects for a time machine advocate do not seem bright.

First, some notions of a time machine — such as Earman et al. (2009) — require that one considers a causally well-behaved spacetime region such that all of its extensions which have "no holes" contain CTCs. This is postulated in order to avoid extensions in which every CTC is split into two or more causally well-behaved timelike curves by appropriate removal of closed subsets from the spacetime manifold. However, arguably there is no "no hole" condition which could fulfill such a task in Kerr spacetime: from the two classes of extensions through Cauchy horizons (extensions which do have CTCs and extensions which are causally well behaved), both of them tend to satisfy or violate the same type of "no hole" conditions. I will illustrate this claim using conditions which require that domains of dependence of achronal subsets cannot be enlarged (such as Minguzzi (2012)), and with a "non-modal" notion of epistemic holes (Manchak (2016)). If so, then intuitions stating that the initial conditions should "bring about" CTCs turn out to be surprisingly hard in cashing out using precisely formulated notions.

Second, I will briefly report on recent improvements on instability of Cauchy horizons and understanding of the singularity structure of rotating black holes. Cosmological blueshift due to de Sitter boundary conditions may prevent curvature blowup for near-extremal Reissner-Nordstrom black holes. However, it has been argued that in the Kerr case, the decay rates of quasi-normal modes respect cosmic censorship (Dias et al. (2018)), and that the Reissner-Nordstrom result is an artifact of the choice of the class of initial data sets (Dafermos and Shlapentokh-

Rothman (2018)); moreover, recent numerical results of Chesler et al. (2018) suggest that for the late times the singularity structure of rotating black holes becomes spacelike. This gives some plausibility to the cosmic censorship hypothesis. But if a form of cosmic censorship holds in classical general relativity, then a time or Malament-Hogarth machine cannot reliably form.

Third, even if some spacetime is a good candidate for a time machine in the context of classical general relativity, requiring compatibility with other physical theories and principles may make it a much worse one in the broader context. I will discuss three arguments of this sort: based on (1) a Generalized Second Law of thermodynamics (Wall (2013)), (2) a form of cosmic censorship due to behavior of the stress-energy tensor in quantum field theory in curved spacetime (Hollands et al. (2019); in this context I will also revisit a more general notion of a quantum compatible non-globally hyperbolic spacetime due to Kay (1992)); and finally, (3) recent arguments by Rovelli (2019) to the effect that time travel to the past is incompatible with standard thermodynamics. I will point out that some of these thermodynamical arguments sensitively depend on the form of causality violations; in particular, they can be avoided in the Goedel spacetime, and (to some extent) in the maximally extended Kerr as well.

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On Surplus Structure Arguments

December 22, 2019

The process of theorizing about the physical world is difficult. This is one reason why scientists and philosophers look to symmetries, which are held to have potential metaphysical and epistemological significance (Brading *et al.* [2017], §5). For instance, suppose one can exhibit that, according to a certain theory, a structure representing a putative feature of the world varies under application of symmetries of the theory—it is surplus structure (Redhead [1975], p. 88). 'Symmetries can be a potent guide for identifying superfluous theoretical structure' (Ismael and van Fraassen [2003], p. 371) as, in their presence, one has reason to believe that, ceteris paribus, the variants of this structure represent no real distinctions in the phenomena that theory represents.

But following this guide is no trivial matter. This is because excess, surplus, or superfluous structure in a theory does not typically announce itself. Proceeding semantically (rather than syntactically), one must first 'generate a set of models rich enough to embed the phenomena, [then] attempt to simplify those models by exposing and eliminating down excess structure' (Ismael and van Fraassen [2003], p. 390) that is idle in how the theory represents the phenomena. Classic examples include Leibniz's static and kinematic shift arguments against absolute space using the Principles of Sufficient Reason and Identity of Indiscernibles,[1] and the use of U(1) gauge transformations to argue against absolute potentials in electromagnetism (Healey [2007]).

Although many authors recognize the validity of arguments roughly of this form, there is still some disagreement as to what that form is, and its own range of validity (Belot [2013]). Dasgupta ([2016]) has however recently brought new analytical focus to the structure of this argument and what it entails for the nature of symmetry thus invoked.[2] He argues that there are two ways in which the surplus structure argument involves epistemic considerations. First, they license inferences from the undetectability of surplus structure in a theory to an observationally equivalent theory, ceteris paribus, in which that structure does not represent any real feature of the world. Second, antecedent analysis of which worldly features are detectable through our faculties of perceptions, prior to and independent of the metaphysics of the theories to which it is applied, must justify what get to count as symmetries of observables.

I agree with the first but part ways with him on the second. In particular, while I agree that it is an Occamist norm to which the surplus structure argument appeals, I also point out several unmet obstacles to defining observational equivalence prior to any interpretation of the theories to which it is supposed to apply. First, it requires an

[1] There is a large literature on this type of argument. See, for instance, Hacking ([1975]), Belot, Belot ([2001, 2003]), Ismael and van Fraassen ([2003]), Dasgupta ([2016]), and references cited therein.

[2] Dasgupta's position is (despite his asseverations to the contrary) very similar to that of Ismael and van Fraassen ([2003]). Nevertheless his presentation deserves the touchstone honor (as I say in what follows) for it is much more perspicuous on the present items of discussion.

implausibly strong a priori epistemology of observation that must be settled prior to any scientific theorizing. Second, because the definition requires applying certain formal conditions to every model of a theory, theories that allow for the occassional non-symmetric model will spoil the symmetries of the others. Instead, I suggest that the concept of observability in theories and the interpretation of those theories proceed hand-in-hand through a process of reflective equilibrium. This account also fits better the historically paradigmatic uses of the surplus structure argument, and how scientists responded to cases in which what was previously thought to be surplus structure was not. As an example, I analyze the 1956 discovery by C. S. Wu and her colleagues of parity-violation in beta decay experiments of cobalt-60. To do so, I develop some elementary formal tools for describing symmetry based on the homomorphisms of structures that preserve certain relations—these relations encode the properties that the symmetry mappings preserve or not. I conclude with some positive remarks (like those of Dasgupta ([2016])) on the scope of surplus structure arguments against the skepticism of Belot ([2013]): the surplus structure arguments do have a generally valid form, as is revealed once we are attentive to certain details about different types of symmetry and the role of isomorphism vis-á-vis representational capacities.

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Several Steps in the Procedure of Hilbert's Axiomatic Method

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Hilbert's axiomatic method, from the procedural point of view, was an analytic method that included, in dynamic unity, synthetic procedures and meta-theoretical reflections. Its main purpose was to find appropriate axioms for given domains of knowledge (in order to present well-structured and reasonably grounded scientific theories for them); however, at same time, its structure allowed for the exploration of alternative domains as well. In my talk, I will present the five general steps of the procedure of Hilbert's axiomatic method: (1) assembling a given domain of knowledge; (2) developing a suitable language for that domain; (3) continuing with a first selection of the axioms; (4) advancing with the logical reduction and with the meta-theoretical inquiry; (5) looking for logically possible alternative theories. Since the method was conceived of as dynamic "tool", the theoretician employing it could flexibly alternate between the stages in order to find appropriate systems of axioms as well as to explore reasonable alternatives. Once understood in such a way, the procedure of Hilbert's axiomatic method appears to greatly resemble the scientific methodology developed by the Andréka-Németi group for the logical foundations of theories in physics.

Beyond Hilbert's Axiomatic Method

In section one, I discuss the significance of three respects in which the Andréka-Németi group 's work in developing the logical foundations of the relativity theories extend Hilbert's axiomatic method. In section two, I suggest another direction of extension.

The Andréka- Németi group's extensions that I consider here are: (1) to have several axiomatic theories to derive the *same* phenomena, as opposed to just having one, (2) to develop theories that fall short of capturing the phenomena of the relativity theories, this is a type of negative counter-factual result and (3) to investigate positive counter-factual information. I shall explain why and how this is important for physics (as opposed to just logic).

The other direction of extension is to think about contradictions between theories using the system of chunk and permeate as developed by Brown.[1] The idea is to cordon off well-behaved, or consistent, chunks of theory. If we reason from one chunk to another, we only allow some information to permeate to the next chunk. In particular, we only allow information or formulas that are consistent with the second chunk to permeate from the first. This is what ensures that we never reason through an inconsistency to prove a theorem. This gives us an interesting way of extending Hilbert's axiomatic method. I explain why this is significant for physics and the unity of science.

Section 1:

Firstly, in developing the logical foundations of the relativity theories, the Andréka-Németi group develop axioms that are formal, precise and sufficient to logically derive the recorded phenomena of the relativity theories. In the course of developing the axioms, they noticed that they were making some choices about the axioms. For this reason, they decided to explore the choices, and develop several axiomatic theories that each logically derive the relevant phenomena. This is significant because it makes it clear what choices are made in developing a scientific theory.

Secondly, in the spirit of reverse mathematics as developed by Harvey Friedman, they also wanted to know what were the weakest axioms that were sufficient to derive the phenomena, and also where the axioms fail, that is, when it is that they are too weak. This extension of Hilbert's axiomatic method, allows us to explore the edge of the theory, by looking at it from the side of failure. Why is negative counter-factual information like this significant? It is important especially when looking for a conceptual path from one theory to another, for example when making the conceptual transition from special to general relativity theory.

Thirdly, there is a more positive or neutral version of counter-factual information where we just posit, or propose, a development in the theory, something not yet encountered. This is important for prediction, and accounting for new discoveries. For example, this might help us to find a way of detecting, or recognising, objects that travel faster-than-light (Andréka et. al. 2012)

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[1] Brown, B. and G. Priest "Chunk and Permeate, a Paraconsistent Inference Strategy, Part I: The Infinitesimal Calculus" *Journal of Philosophical Logic*. 33(4), 379-388.

Section 2:

Brown and Priest (2004), developed a method of “staying consistent” while reasoning from inconsistent premises. They call the inference strategy “Chunk and Permeate”. The idea is to divide proofs into internally consistent chunks. We then allow only some information to overlap between chunks. The idea is that some information permeates from one chunk to another. We do this frequently when reasoning in science; Brown and Priest just make it explicit.

The relevance of this for the work of the Andréka-Németi group is to justify their extension of Hilbert’s axiomatic method. The justification is important because if we assemble all of the information that the Andréka-Németi group have learned about the relativity theories, then we would observe that it is inconsistent. This is obvious if we think of the negative counter-factual information discussed above together with the information in the successful axiomatic theories. In a classical or intuitionist logic, the inconsistency brings triviality: all formulas that can be written in the language would be a part of the theory, that is, all formulas *and their negations*. This would be a disaster for a scientific theory.

However, we do not need to be alarmed. Very naturally, since it is standard scientific and mathematical practice, they never *use* the contradictions to derive formulas. To demonstrate the legitimacy (consistency in reasoning) of Hilbert’s axiomatic method extended by the Andréka-Németi group, we can use Brown and Priest’s inference strategy.

I provide an example from the work of the Andréka-Németi group.

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Philosophy of Howard Stein: Prospects for Possibilism

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In his discussion of the ontology of time with respect to physical geometry, Hilary Putnam first addresses himself by appealing to the authority of "the man on the street" to whom he ascribes the position that

P. All (and only) things that exist now are real.[1]

He then pronounces three assumptions clarifying the ontological commitments necessary to sustain this position before proceeding to assume Special Relativity as his domain of discourse. In reference to the space-time diagram of the intersecting worldlines of two observers, Putnam demonstrates that the events comprising the "now" of one of the observers correspond to the events in the "future" of the other and that, therefore, future events should be regarded as real.

In response to Putnam's argument, Howard Stein identifies a misapplication of the theory's intrinsic geometry to the argument and criticizes Putnam's failure to recognize how Special Relativity invalidates all pre-relativistic notions of simultaneity.[2] In addition, Stein roundly condemns Putnam's deductive methodology on the ground that he draws a correct conclusion from philosophical principles wholly inappropriate to the scientific context in which they are employed. In a more recent paper, Simon Saunders revisits the Putnam vs. Stein debate to conclude that "the man on the street's" view of time can finally be laid to rest as incompatible with Special Relativity.[3]

Saunders' verdict is an open invitation to reflect on how the reality of future events is to be understood in the aftermath of the Putnam vs. Stein debate. Abandoning the belief that only the present is real should put other "man on the street" type of beliefs about the nature of time under scrutiny. For instance, there are people who believe in fate, in synchronicity or some form of guiding principle under which some or all of their future experiences have been predetermined. How do these beliefs fare in the context of Special Relativity? Is there anything in the theory that might explain why our recollection of the past is different from our speculation about the future? To set the stage for

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1. Hilary Putnam, "Time and Physical Geometry," *Journal of Philosophy* 64, no. 8 (1967): p.240.

2. Howard Stein, "On Einstein–Minkowski Space–Time," *Journal of Philosophy* 65, no. 1 (1968): 5–23.

3. Simon Saunders, "How Relativity Contradicts Presentism," *Royal Institute of Philosophy Supplement* 50 (2002): p.13.

the discussion that follows, we look at the opening lines of T.S Eliot's *Burnt Norton*:[4]

> Time present and time past
> Are both perhaps present in time future
> And time future contained in time past.
> If all time is eternally present
> All time is unredeemable.
> What might have been is an abstraction
> Remaining a perpetual possibility
> Only in a world of speculation.
> What might have been and what has been
> Point to one end, which is always present.

With ambiguity characteristic of Eliot's style, the poetic voice muses on the ephemerality of time before delving into the inner world charged with disjointed images from the author's memory that constitutes the body of the poem. The lesson to be drawn from this preambulatory passage is that the experience of time cannot be separated from our role as living witnesses of the *becoming* of the world.

This notion of *becoming* is central to the main goal of this essay, which is to demonstrate that even in Special Relativity the past is distinguished from the future in that the former has become determinate while the latter is only apprehended in terms of possibilities. In the next section I will introduce the terminology of *presentism* and *eternalism* under which the Putnam vs. Stein debate is currently understood. This will be followed by a logical analysis of Putnam's premises and Stein's criticisms before clarifying why Putnam's premises are invalidated by the intrinsic geometry of Einstein-Minkowski space-time. Finally, we will return to the notion of *becoming* by discussing the indefiniteness of the truth-value of future events.

I Presentism and Eternalism

In premise P we encountered the presentist position: the belief that only the present moment exists. Events cease to exist the moment they become past and those that are future have yet to come into existence. For the presentist the world is a Heraclitean fire consuming itself perpetually in the recreation of the momentary now. But if time is "eternally present," Eliot tells us, then "all time is unredeemable." The word "unredeemable" might be interpreted as signifying that for the presentist, there is no way to regain the past and no way to bring about the future other than through *becoming*. The ontological sparseness of this view is strongly contrasted by its dialectical opposite.

For the *eternalist* the past and future are as equally endowed with existence as the momentary now. This view finds its most natural expression in the block-universe: a 3D cube on which the vertical direction represents time while each horizontal 2D slice corresponds to the configuration of 3D Euclidean space

4. Thomas S. Eliot, "Burnt Norton," in *20th Century Poetry & Poetics*, ed. Gary Geddes (Oxford University Press, 1973), p.89.

at any given instant.[5] In such a block, an observer's "earlier" and "later" are to be regarded merely as a matter of perspective; moving from "earlier" to "later" is akin to being displaced from "here" to "there." This picture of a block universe is very appealing in classical Lagrangian mechanics, where by application of a variational principle one may construct the equations of motion for a conservative system. Indeed, by taking the difference between the kinetic and potential energy of the system, one can solve a partial differential equation to obtain solutions of its spatial coordinates as unique functions of a parameter t. Then, by application of Noether's Theorem, we recognize that the equations of motion are fully time-reversible: the spatial configuration at any time in the dynamic evolution of the system can be specified with the same facility that one can start and stop an animation in a computer screen with a click of the mouse.[6]

However, the beautiful symmetries of Noether's theorem are applicable to a very small and highly idealized class of physical systems. A potential energy cannot usually be specified for dissipative systems, which introduces an inherent notion of *irreversibility*. In thermodynamics, this problem is known as the *arrow of time*, an extensive treatment of which is beyond the scope of this paper. Moreover, and unlike the point-particles and rigid bodies of Lagrangian mechanics, human beings are endowed with both the faculties of memory and imagination. An eternalist committing to the existence of future events should be able to explain why, despite having reliable records of the past states of some systems, their future outcomes may be nonetheless undetermined. To wit: whereas I know with certainty that it rained earlier today, I can only calculate probabilities for whether it will rain tomorrow or not.

Thus, while the presentist has to find a satisfying explanation for the Heraclitean fire of perpetual becoming, the eternalist must explain the Parmenidean unity of all time. A person can remember past events with varying degrees of reliability, but there is no such thing as remembering the future. Thus, in committing to the existence of the future despite the possible indefiniteness of outcomes, the eternalist must identify the illusory aspects of his experience that prevent him from having full certainty about the truth-value of future events. Indeed, the "perpetual possibility" that Eliot's poetic voice identifies in relation to "what might have been" is equally applicable to *what might still become*. The fact that we are able to devise a "world of speculation" as we witness the becoming of the world characterizes *possibilism*, the intermediate view between presentism and eternalism. A proper discussion of possibilism, however, will be deferred to the aftermath of the Putnam vs. Stein debate, which I shall now address.

II Putnam's Principle

By identifying the proposition P as presentist, we now have the necessary terminology to reconstruct the debate in full. In doing so, however, the reader should keep in mind that neither Putnam nor Stein use either of these terms in their

5. Steven Savitt, "Being and Becoming in Modern Physics," in *The Stanford Encyclopedia of Philosophy*, Fall 2017, ed. Edward N. Zalta (Metaphysics Research Lab, Stanford University, 2017), p.3.

6. John R. Taylor, *Classical Mechanics* (University Science Books, 2005), p.272.

papers. Putnam's goals for his essay are two-fold. First, he aims to show that the application of his principles to Special Relativity requires an acceptance of the reality of some future things, which can be extended "to show that all future things are real ... and likewise that all past things are real, even though they do not exist now."[7] Second, he concludes "that contingent statements about future events already have a truth value."[8] On both regards, then, Putnam's paper should be seen as advocating for eternalism in light of Special Relativity. Stein's objective is not to defend presentism or eternalism, but rather to reconstruct the rudiments of Special Relativity with the intent of denouncing Putnam's misapplication of the theory's geometry to arrive at his conclusion and to further accuse him of the "lowering of critical standards in philosophical discourse" which "precludes understanding and is the death of philosophy."[9] Is this brutal criticism justified? To find out, we must examine in detail Putnam's argument.

To clarify what he intends by P, Putnam offers three assumptions:

I. I-now am real (Of course, this assumption changes each time I announce that I am making it, since 'I-now' refers to a different instantaneous 'me'.)

II. At least one other observer is real, and it is possible for this other observer to be in motion relative to me.[10]

These two assumptions are merely definitional and are in and of themselves not controversial. The third assumption, which Putnam calls the principle that "There Are No Privileged Observers" is of a markedly different standing—it captures Putnam's philosophical commitment throughout his paper despite the complete inadequacy of any attempt to articulate it in Special Relativity. It reads:

III If it is the case that all and only the things that stand in a certain relation R to me-now are real, and you-now are also real, then it is also the case that all and only the things that stand in the relation R to you-now are real.

One cannot afford to be too careless in making sense of this principle. It is clear that Putnam intends the relation R to be understood as a formal logical proposition.

In hopes of clarifying Putnam's assumptions, I shall express them symbolically as follows:

$$\exists m(mN) \tag{1}$$

$$yN \wedge yVm \tag{2}$$

Here m denotes me as an observer such that my being "real-now" is captured by satisfying the predicate N. Now, statement II depicts you as the observer y such that you and me satisfy the relation V of moving at some velocity relative to each other. Putting together I and II we arrive at Putnam's principle:

$$\forall\theta(\theta Rm \wedge yN \rightarrow \theta Ry) \tag{3}$$

7. Putnam, "Time and Physical Geometry," p.246.
8. Ibid., p.247.
9. Stein, "On Einstein–Minkowski Space–Time," p.20.
10. Putnam, "Time and Physical Geometry," p.248.

The attentive reader will note that my formulation of (3) is different from both Stein's formulation and that of Saunders. In a way, this is to be expected because Putnam's language in the statement of the principle has an ambiguity between the reality of the observers and the reality of the things standing in relation to them. Noting first the use of logical connectives in III, the order in which "if", "and", and "then" appear makes it clear that the principle has the form of a syllogism, namely $p \wedge q \rightarrow r$. Now, taking the phrases "all and only" and "real" to denote universal quantification over the things θ, it is a matter of logic to identify p as "the things that stand... in relation R to me-now" and identifying r as "the things that stand in the relation R to you-now." While there is no equivocation in how to represent p and r syntactically, there is an inherent ambiguity in deciding how to represent the identification of q with "you-now are also real." The existence of another observer on its own implies no presuppositions about other things, while the predicate "now" is a direct reference to Putnam's definition of what constitutes an observer by assumption I. As we shall see in the next section, the change in meaning between "me-now" and "you-now" as we move from pre-relativistic theory to Special Relativity is at the heart of Putnam's confusion.

In his discussion of Putnam's principle, Stein posits that we should take the relation R to entail "a is real to b" for any two points in space-time, which allows him to advance the proposition that if things are real to me and you are real to me, then things are real to you.[11] Clearly, this statement can be written in the form

$$aRb \wedge bRc \rightarrow aRc \tag{4}$$

By noting that reflexivity and symmetry are implied—b is real to b and b being real to a surely requires the converse to hold—we can identify the relation R to specify an equivalence class. However, Stein expresses his skepticism of the validity of this formulation of the principle:

> ...one easily sees that in effect what [Putnam] calls the principle of No Privileged Observers just requires R to be an equivalence relation. But such a requirement has in fact no connection with the privilegedness of observers; and it is moreover entirely inappropriate to Einstein-Minkowski space-time—in which (unlike pre-relativistic space-time, with its temporal decomposition) *there are no intrinsic geometrical partitions into equivalence classes at all, besides the two trivial ones*: that into just one class (all of space-time) and that into classes each consisting of a single point[12]

The problem with Stein's criticism is not with his analysis of the intrinsic geometric properties of Einstein-Minkowski space-time, but rather with the misattribution to what Putnam intents the relation R to mean. Putnam clearly interprets "the relation R to be the relation of simultaneity"[13] for both the pre-relativistic case and in Einstein-Minkowski space-time. Thus, the right reading of (4) is that if things are simultaneous to me-now and you-now are simultaneous to me, then things are simultaneous to you-now.

11. Stein, "On Einstein–Minkowski Space–Time," p.19.
12. Ibid.
13. Putnam, "Time and Physical Geometry," p.241.

Returning to my own formulation of (3), it should be clear that treating the notion of reality as synonymous to existential quantification is a better formulation of Putnam's principle than treating reality as a relation because it disambiguates Putnam's intended usage of R from Stein's interpretation. The reality of things in the bound variable θ depends on the proposition inside the parenthesis being satisfied. In order for my formulation to define an equivalence class, it has to be written in the form

$$\forall\theta(\theta Rm \wedge mRy \rightarrow \theta Ry) \tag{5}$$

which is only satisfiable in the case that

$$yN \leftrightarrow mRy \tag{6}$$

The question of the validity of III has thus been reduced to the question of whether you-now are real if and only if you are simultaneous to me. Putnam's requirement that the things θ be understood in tenseless language is satisfied by (5) in the sense that all things θ are being quantified over irrespective of whether they are past, present or future. However, Putnam stipulates that "R must be restricted to physical relations that are supposed to be independent of the choice of a coordinate system (as simultaneity was in classical physics) and to be definable in a 'tenseless' way in terms of the fundamental notions of physics."[14] Whether this criterion obtains will not be determined by logic, but rather by the geometry of Einstein-Minkowski space-time.

III Nowness and the Relativity of Simultaneity

In pre-relativistic physics, the validity of Putnam's principle can be easily ascertained: the instant defining my now identifies all things standing in simultaneous relation to me-now. If you-now are also real, it immediately follows that you are simultaneous to me *because there is only one possible foliation of space-time into simultaneity slices for all observers.* Your reality "now" immediately requires (6) to be satisfied, thereby making (3) equivalent to (5). This observation is in agreement with Stein's analysis of pre-relativistic space-time, where he posits that for two space-time points a and b, it is "easy to show that a is in the past of b if and only if b is in the future of a." By taking "being in the past of" to define an asymmetric and transitive relation, Stein defines a "chronological ordering" C such that if neither aCb nor bCa obtain, it follows that a must be identical to b. Formally, this is a partial ordering $<N_t, \leq>$ on the set of time-slices in pre-relativistic space-time: if we take N_t to define the "now" of a given time-slice, (6) is satisfied such that (3) can be reduced to (5) to assign reality to all things simultaneous to both me-now and you-now every time the shared "now" is instantiated. However, this partial-ordering is not globally satisfied in Einstein-Minkowski space-time because (6) is not satisfied in general.

In the previous section, I pointed out that while Stein misattributed what Putnam intended the relation R to mean, his analysis of the intrinsic geometric properties of Einstein-Minkowski space-time correctly identifies Putnam's mistake in making the attempt to situate (3) in the context of the theory of

14. Putnam, "Time and Physical Geometry," p.241.

Special Relativity. At the beginning of his paper, Stein provides the following definition:

> Space-time is a four-dimensional real affine manifold, given together with a class of non-degenerate inner products on the associated vector space: this class is closed under multiplication by nonzero real numbers, and is generated (under this condition) by any of its members; the members of the given class have index of inertia 1 or 3.[15]

Some rudiments of how this definition is implemented in practice are in order. First, the difference in index of inertia is readily seen in the matrix representation of the Minkowski metric, which may be written as[16]

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

The non-degenerate inner-products are expressed in the summation convention as follows

$$\eta_{\mu\nu} du^{\mu} dv^{\nu} = u^0 v_0 + u^1 v_1 + u^2 v_2 + u^3 v_3 \tag{8}$$

where the index 0 is called the time-like component of the vector and the indices 1,2,3 are called space-like components. In particular, when the vectors are regarded as infinitesimal displacements in the real affine manifold, denoted as du^{μ} and dv^{ν}, one arrives at the crucial result that

$$\eta_{\mu\nu} du^{\mu} dv^{\nu} = ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{9}$$

Equation (9) specifies the line-element ds^2, also known as the space-time interval, of the theory. For any two events represented by vectors indexed with coordinates μ and ν, the inner product specified by the line-element is guaranteed to give the distance between them in the manifold in a way that is invariant under whatever choice of coordinates one might make. The second equality in (9) expresses the interval in Cartesian coordinates with the speed of light set to $c = 1$, which is the only case that we need to consider here. Finally, Special Relativity classifies the space-time interval between any two events into three distinct classes:

- events for which $ds^2 < 0$ are said to be *time-like* separated
- events for which $ds^2 > 0$ are said to be *space-like* separated and
- events for which $ds^2 = 0$ are said to be *light-like* separated.

In accordance with Stein's reconstruction of Einstein-Minkowski space-time, the class of points in time-like separation from a is disconnected into two components: the past light-cone and the future light-cone of a. It is only with respect to a given *point* that the chronological ordering of events can be specified. All events contained in the past light-cone of a constitute its past while all the

15. Stein, "On Einstein–Minkowski Space–Time," p.6.
16. James B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Addison-Wesley, 2003), p.135.

events contained in the future light-cone of a define its future. This leaves unaccounted the events light-like separated from a, which the theory identifies as defining the boundaries of a's light-cone, and the space-like separated events, whose interpretation is responsible for Putnam's confusion.

In attempting to export his assumptions to the arena of Special Relativity, Putnam fixes a geometric scenario in which you-now and me-now cross paths on a space-time diagram while traveling at near-light speed (nevermind the physical implausibility of accelerating a human anywhere *near* those speeds). He warns us that we "cannot take the relation of simultaneity-in-my-coordinate system to be R" without violating (5), but should rather "take R to be the relation of simultaneity-in-the-*observer's*-coordinate-system."[17] This is just plain wrong. In Special Relativity no choice of coordinates is privileged over any other *because the physical content of the theory is preserved by the space-time interval*. Therefore, if mRy is to be satisfied at all, it must be the case that the space-time interval between me-now and you-now is space-like: what coordinates both observers are using is irrelevant. Indeed, this is very neatly illustrated by the partitioning of Einstein-Minkowski space-time into time-slices introduced by Saunders in his discussion of Putnam's principle.[18] Given the manifold M, it may be partitioned into slices $\{M_t\}$ indexed by some parameter t such that

$$R = \{< a, b > | a \in M_t \wedge b \in M_t\} \tag{10}$$

Taking a to be the event of me-now, then mRy is satisfied if you-now are located in some event b that is space-like separated from a. Then the set of all events c that are space-like separated from a at that t, $\{< a, c > | a \in M_t \wedge c \in M_t\}$, defines a simultaneity slice for a. However, unlike the pre-relativistic case, there is nothing in this construction that makes a choice of M_t be unique. In fact there are *infinitely* many ways in which space-like surfaces can be defined relative to a point, which emphasizes that no universal concept of simultaneity can ever be defined in Special Relativity.[19] Moreover, as pointed out by Stein, Einstein's analysis reveals that "our procedures of spatio-temporal measurement single out—de facto—a particular state of motion" such that "the quantities we measure are, intrinsically, relative to a time axis."[20]

Because Putnam's construction requires you-now to be moving with some velocity relative to me-now, your time axis is differently oriented from mine such that our foliations of Einstein-Minkowski space-time into time-slices cannot be identical. To see this, suppose that the displacements in (9) are not infinitesimal. Then, denoting the coordinates of you-now by the subscript b and the coordinates of me-now by the subscript a we have that

$$(\Delta s)^2 = (s_b - s_a)^2 = -(t_b - t_a)^2 + (x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2 \tag{11}$$

All that is required for you-now and me-now to be at space-like separation is that $(\Delta s)^2 > 0$. One way for this to happen is the case that $t_b - t_a = 0$, which would give

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \tag{12}$$

17. Putnam, "Time and Physical Geometry," p.242.
18. Saunders, "How Relativity Contradicts Presentism," p.6.
19. James B. Hartle, "The Physics of Now," *American Journal of Physics* 73 (February 2005): p.4.
20. Stein, "On Einstein–Minkowski Space–Time," p.12.

but much more generally any inequality of the form

$$(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 > -(\Delta t)^2 \tag{13}$$

will do. The key thing to note here is that there is nothing transitive about (13)! Thus, the fact that the coordinates of some thing θ at point c have a space-like interval to me-now in no way requires θ to also be space-like separated to you-now. In fact, it is perfectly plausible to find a θ at c such that it is space-like separated to you-now but time-like separated to me-now.[21]

Putnam arrives at his conclusion by requiring the event at c to lie in the future light-cone of me-now and claims that because θ is simultaneous to you-now it is real and that therefore I must regard θ to be real to me-now even though it lies in my future. This conclusion, although it correctly implies some form of eternalism, cannot be logically deduced from its premises. Putnam's principle requires (6) to be satisfied to reduce (3) to (5), but by equation (9) it will always be possible to find an event that is space-like separated to one of the observers but not to the other. Therefore, the principle that there are No Privileged Observers cannot be satisfied in Einstein-Minkowski space-time. Every single point has its own unique now—one of the two equivalence classes identified by Stein—such that presentism can no longer be defended unless one subscribes to "a peculiarly extreme (but pluralistic!) form of solipsism".[22]

IV Possibilism Proper

The other equivalence class identified by Stein in his rejection of Putnam's principle is the totality of space-time. What should we take it to mean? If we want to be realists about Einstein-Minkowski space-time, we should accept the correctness of Putnam's conclusion despite its failure to follow logically from its premises: we must grant that the events lying in the future light-cone of an observer are real. At this point, however, we encounter the challenge that I laid out for the eternalist at the end of the first section: given that future events are real, how come our knowledge of them is *indefinite*? I will argue in this section that the answer to this question is best captured by *possibilism* once it has been recast in a way compatible with Special Relativity.

In the second part of his paper, Putnam opens the question of the determination of the future by the past by revisting an ancient debate first considered by Aristotle. The question is simply whether there will be a sea-fight tomorrow, or in a whimisical turn of phrase of Putnam's, whether there will be a *space*-fight tomorrow. In strict logical terms, a proposition such as

S. There will be a space-fight tomorrow.

is either true or false. From a purely epistemological point of view, the best way to find out the outcome is simply to wait, but the question Putnam is interested in is whether such an outcome has already been determined *today* for me-now, that is, whether its truth-value has been determined *tenselessly* irrespective of a given observer's knowledge of the future outcome. He then tells us that for Aristotle,

21. Taylor, *Classical Mechanics*, p.627-627.
22. Stein, "On Einstein–Minkowski Space–Time," p.15.

> there is a fundamental difference, ... between the past and the future, viz., that past events are now determined, the relevant statements about them have now acquired truth values which will "stick" for all time; but future events are undetermined, and at least some statements about them are not yet either true or false.[23]

By this reasoning, the past and present are unambiguosly distinguished from the future because the truth-value of logical propositions with respect to events has become determinate. According to this view, the truth-value of whether it was going to rain today was *undecided* between true and false and *became* true the moment the first drops of rain started to fall against my window-sill.

In his treatment of Putnam's paper, Saunders determines that by "shifting to the question of what statements have truth-values it is surely intended that we include statements referring to past events as well as to present ones," which leads him to define possibilism as the thesis that "only the present and the past is real."[24] Defined in this way, however, possibilism is unsustainable. The reality of past, present and future events should not be confused with the question of whether logical statements about these events have a definite truth-value, *especially* after we incorporate the lessons from the previous sections into our ontology of time. Therefore, Saunders' definition of possibilism should be ammended to claim that "only the present and past have been determined," but even this has to be sharpened further.

In a way analogous to how the pre-relativistic notions about nowness and simultaneity are invalidated by Special Relativity, the notion of the defineteness of the truth-value of statements must be revaluated in a way that is compatible with the intrinsic geometric relations of the theory. In pre-relativistic space-time, with an absolute standard of time, there is a global chronological ordering of all time-slices $\{N_t\}$. In this context, the truth-value of all statements should be no different to you-now than it is to me-now because (6) is satisfied. It doesn't matter whether you were around to see the raindrops falling against my window-sill: the fact that I saw it happen makes it true for all observers at all times. Similarly, my being too far away from the site of the alleged space-fight should not undermine your ability to verify with your own eyes whether it happens tomorrow or not. The determination of the truth-value of a future event for one observer is sufficient to determine it for all other observers. This is no longer the case in Einstein-Minkowski space-time.

In this regard, Stein's analysis proves to be particularly insightful. First, he epitomizes the notion of *becoming* in Special Relativity through the statement that

> For an event—a man considering, for example—at a space-time point a those events, and only those, have already become (real or determinate), which occur at points in the topological closure of the past of a.[25]

where by "topological closure" he means the union of a with its past light-cone. He follows this assertion with the observation that

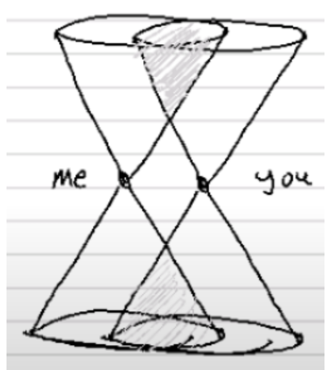
23. Putnam, "Time and Physical Geometry," p.244.
24. Saunders, "How Relativity Contradicts Presentism," p.7.
25. Stein, "On Einstein–Minkowski Space–Time," p.14.

> "having or not having a truth value," in this question, must be understood classically to mean "at a given time" (the puzzle about the [space]-fight tomorrow is whether there is a definite truth value *today*); but "at a given time" is not a relativsitically invariant notion, and the question of definiteness of truth value, to make sense at all for Einstein-Minkowski space-time, has to be interpreted as meaning "definiteness at a given space-time point (or event)"—to be vivid: "definiteness for me now."[26]

This notion of the determination of the truth-value of an event being definite only for me-now, does indeed come off as somewhat solipsistic. I claim, however, that in can be interpreted epistemologically in terms of an observer's ability to *learn* about the outcome of a given event. If you-now see the space-fight happening and want to tell me about it, you will have to send me the news as a message so that my knowledge about its truth-value will only *become* definite the instant I receive the signal of your message through my past-light cone. Stein's analysis captures this idea through the concept of *contemporaneity* in Einstein-Minkowski space-time, which he defines in the following way:

> two such processes may be said to be contemporaneous if part of each other is past to the part of the other—in other words, if mutual influence ("communication") is possible between them.[27]

I find this idea to be so important that I include a schematic of it below:



In this figure, the points at which you-now and me-now are located are space-like separated. However, our past light-cones intersect so that the shaded region corresponds to the events that are contemporaneous to each other. Only the events in this region have a truth-value that is definite for both observers. The events that lie in the past light-cone of you-now but not in the shaded region only have a definite truth-value for you-now, but not for me-now. Naturally, the same holds for the events that are in the past light-cone of me-now but not of you-now. Therefore, the image illustrates my claim that in Einstein-Minkowski space-time, past events that have a definite truth-value for one observer may not necessarily have a definite truth-value for other observers.

26. Stein, "On Einstein–Minkowski Space–Time," p.14-15.
27. Ibid., p.15.

Now, what are we to make of the shaded area in the future light-cone of the observers? In this case, Special Relativity tells us that the events lying in the intersection of the future light-cone will be those that can be *causally-influenced* by both you-now and me-now. Evidently, whether events in the future of both observers can be causally influenced by them does not decide whether the truth-value of statements that can be made about them is determined *tenselessly*. Thus, if Putnam's second conclusion that "the 'tenseless' notion of existence (i.e., the notion that amounts to 'will exist, or has existed, or exists right now') is perfectly well-defined,"[28] it must do so in a way unrelated to the definiteness of the truth-value of events for observers.

In conclusion, Special Relativity does indeed rule out presentism as the correct metaphysical view of the ontology of time, leaving possibilism and eternalism as alternatives. On the one hand, the eternalist has to understand the existence of past and future events as encompassing events that are currently unknown, or may even be unknowable in principle. On the other hand, the possibilist has to accept the reality of future events if she wants to be a realist about Special Relativity. For her to accept the reality of past events while rejecting the existence of future events would be inconsistent with the intrinsic geometry of the theory. Therefore, possibilism should be philosophically understood as a view about how the truth-value of future events is decided by the *becoming* of the world with respect to individual observers. Finally, it seems that the question of whether there is anything that might explain why our recollection of the past is different from our speculation about the future is not decidable by the intrinsic geometry of Special Relativity. The answer to this question is an open problem for both physics and philosophy, and its resolution may be someday determined by a successful formulation of Quantum Gravity, or some other hitherto as of yet unknown theory.

word count: 5757

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What exactly does the special principle of relativity state? A discussion of Einstein's 1905 paper

Extended abstract

While there is a longstanding discussion about the interpretation of the extended, general principle of relativity, there seems to be a consensus that the *special* principle of relativity (SPR) is absolutely clear and unproblematic. However, a closer look at the literature on relativistic physics reveals a more confusing picture. This talk will attempt to illustrate this situation by discussing how Einstein uses the SPR in his 1905 paper [1]. It will be pointed out that Einstein applies three different versions of the SPR—three different statements with different physical content. It will be shown how each of the three versions is problematic in its own terms, and, more importantly, how they are manifestly nonequivalent, two of them being even contradictory together. Along the way, our analysis will lead us to pose many obvious, but not obviously answerable, questions about the precise meaning of the SPR.

The first version we shall consider is the SPR as applied in the magnet-conductor thought experiment, by which Einstein famously begins, and motivates, his analysis ([1], 37). There are two customary interpretations of the magnet-conductor scenario. On one account the magnet+conductor system in two different states of overall uniform motion is described from one single inertial frame—in one state the magnet is at rest and the conductor is in motion, in the other one the magnet moves and the conductor is at rest, relative to the given inertial frame in question. On the other account the system's overall state of motion is fixed and one compares its behavior as seen from two different internal frames—one co-moving with the magnet, the other one co-moving with the conductor. It will be shown that on either of these interpretations Einstein's claim according which "the observable phenomenon [the induced current] here depends only on the relative motion of the conductor and the magnet" is only true in an approximate sense, in the non-relativistic limit of $v/c \to 0$, where v is the relative velocity of the magnet and the conductor. It is the relativistic effects and transformation laws derived by Einstein himself in the 1905 paper that render his observations on the magnet-conductor case, by which he motivates the SPR, invalid. Is Einstein's relativity thoroughly inconsistent? Or should the SPR be understood in a different way?

Many will hold it should. For many will take it as obvious that what the SPR actually requires is not that physical quantities (among them the induced current) should have the same values in different frames, but rather that they should vary together so that the functional relationships among them remain the same in all inertial frames of reference. In other words, the physical equations which these quantities obey (among them Faraday's law of induction and Lorentz's force law describing the magnet-conductor case) must be *covariant*. This reading, as a second version of the SPR, also finds support in the 1905 paper. It is the requirement of covariance that Einstein uses when deriving the transformation laws of the electric and magnetic field strengths in the electrodynamical part of the paper ([1], 51–53). Here we will point out a severe ambiguity in the notion of covariance that is manifest in Einstein's treatment. Einstein talks about the covariance of Maxwell's equations but when he actually does the calculations he only writes down two of them, the Ampère–Maxwell and Faraday's law. These two equations, however, are *not* covariant separately—expressing them through the Lorentz transformation laws of the kinematic and electrodynamic quantities one does *not* receive equations of the same form as the original ones. It is only when transforming them together with the other two Maxwell equations that one receives equations of the same form. What is true of the Ampère–Maxwell and Faraday's laws separately is that they hold good in all inertial frames due to the covariance of the *whole system* of Maxwell's equations taken together. This raises the question: if the SPR means covariance, *whose* covariance should it be taken to be? What is so special about the whole covariant system of Maxwell's equations, by contrast with other non-covariant equations, with regard to what the SPR is meant to say? Or should the SPR be relaxed to the condition, weaker than covariance, that equations of the same form must *hold good* in all inertial frames? Without the full-blown covariance requirement, however, it is not possible to arrive at the transformation laws of the field strengths in a way Einstein supposed to. On the other hand, many physical equations fail to satisfy even the weaker condition; nevertheless, the SPR just as well seems to apply to them, as the third version of its application in Einstein's paper demonstrates.

As the third variant we will consider the way Einstein applies the SPR when deriving the equation of motion for the moving point charge in the closing section of the 1905 paper ([1], 61–62). Here Einstein compares *two* situations: one in which the charge is at rest and one in which it is in motion relative to a given inertial frame. He takes the SPR to say that the equations describing the second situation expressed in terms of the co-moving frame must have the same form as the equations describing the first situation expressed in terms of the original frame. We will make three remarks about this understanding of the SPR. First, the equation Einstein talks about—Lorentz's equation for a stationary charge, $m\mathbf{a} = q\mathbf{E}$—is *not* covariant, and does *not* even hold good in every inertial frame, only in the one where the particle is at rest. Moreover, the condition Einstein requires here doesn't seem to follow even from the covariance of the full-fledged relativistic Lorentz equation of the particle; for there is no way to refer to the specific situations involved only in terms of the covariance of a single equation. Secondly, we will compare the charge-moving-in-the-field scenario with a non-relativistic particle moving a viscous medium, and contemplate about the differences of how Einstein's condition applies in the two cases. Thirdly, we will point out that if Einstein had applied this third version of the SPR in the magnet-conductor thought experiment, he would have realized that the observation he makes in the beginning of the paper, again, by which he motivates his whole analysis, is not correct.

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When generalised definitional equivalence implies definitional equivalence

Different strands of research in the theory of definitions and interpretability have led to equivalent generalisations of the classical Tarskian notion of definitional equivalence. Andréka, Madarász, and Németi (2008) have generalised definitional equivalence in a way that allows to define new sort symbols in many-sorted first-order theories (e.g., quotient sorts and product sorts). Barrett and Halvorson (2016) have re-discovered this notion of generalised definitional equivalence under the name 'Morita equivalence'. It is natural to suspect that Morita equivalence coincides with the notion of generalised bi-interpretability used by model-theorists, which allows many-sorted interpretations that are multi-dimensional and non-identity preserving (Halvorson, 2019). As Halvorson reports, the two notions indeed coincide.

In this talk, I explore under which conditions these generalisations go beyond the classical notion of definitional equivalence. More precisely, I address the question for which kind of theories Morita equivalence coincides with definitional equivalence. I present first steps towards answering this question. More specifically, I present work towards a proof of the following

Conjecture 1. *If T and T' are single-sorted sequential theories with strong elimination of imaginaries and there is a generalised bi-interpretation between T and T', then T is definitionally equivalent to T'.*

Sequential theories with strong elimination of imaginaries are, roughly speaking, theories where all imaginary elements of their models (i.e. objects that are results of abstraction) can be identified with some of their real elements. More precisely:

Definition 1. T has elimination of imaginaries iff for every $L(T)$-formula $\phi(\bar{x}, \bar{y})$ such that T proves that $\phi(\bar{x}, \bar{y})$ defines an equivalence relation, there is an $L(T)$-formula $\varepsilon(\bar{x}, \bar{z})$ such that $T \vdash \forall \bar{y} \exists ! \bar{z} \forall \bar{x} (\phi(\bar{x}, \bar{y}) \leftrightarrow \varepsilon(\bar{x}, \bar{z}))$.

Intuitively, theories with elimination of imaginaries are so rich that one can define an abstraction operator for any equivalence relation.

Definition 2. T has strong elimination of imaginaries iff T has elimination of imaginaries and for every $L(T)$-formula of the form $\bar{x} = \bar{y}$ (i.e. $x_1 = y_1 \wedge \ldots \wedge x_n = y_n$), there is an $L(T)$-formula $\varepsilon(\bar{x}, z)$ such that $T \vdash \forall \bar{y} \exists ! z \forall \bar{x} (\bar{x} = \bar{y} \leftrightarrow \varepsilon(\bar{x}, z))$.

The point here is that z is a single variable rather than a sequence of variables. This means that tuples exist in theories with strong elimination of imaginaries.

Many theories have strong elimination of imaginaries, e.g. when they contain a substantial amount of mathematics.

If Conjecture 1 can be proved, this would have two notable consequences.

1. It would yield an extension of the Friedman-Visser Theorem (2014), which specifies the conditions under which bi-interpretability implies definitional equivalence.

2. It would show that, for a rather large class of theories, the new criteria of Morita equivalence or generalised definitional equivalence does not make a difference when compared to the classical criterion of definitional equivalence. So, the criterion of Morita equivalence would only be needed when it comes to theories that do not contain a substantive amount of mathematics.

To approach Conjecture 1, it helps to reduce it to a somewhat simpler conjecture that uses concepts from categorical model theory, namely:

Conjecture 2. *If T and T' have equivalent syntactic categories, then T and T' are bi-interpretable via simple interpretations.*

Proposition. *Conjecture 2 implies Conjecture 1.*

So, all that is left to show is Conjecture 2. A proof sketch for Conjecture 2 already exists. I hope to report a full proof of the conjecture at the meeting.

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The Logic of Logical Positivism

Three attempts to formalize, in logic, some of its central notions

(Introduction)

In a 1967 article, John Arthur Passmore (1914-2004) announced that: *"Logical positivism… is dead, or as dead as a philosophical movement ever becomes. But it has left a legacy behind"* [Passmore, J. A. *Logical Positivism.* In P. Edwards (Ed.). The Encyclopedia of Philosophy (Vol. 5, 52-57). New York: Macmillan.]

The purported 'death' of this school of thought is mainly due to the fact (realized by the logical positivists themselves, also called logical empiricists) that they have not been able to reconcile two of the basic tenets of their philosophical theory of how we have to look at science.

One of the precepts was – based on methodological considerations – that scientific theories should, at least for the aims of meta-scientific analysis, be formalized in logic, *much* like mathematical theories (of natural numbers or of sets, etc.). The other precept was that the concepts or 'terms' proper of a (formalized) scientific theory have to be 'meaningful' in two senses. First, they must not be mathematical terms, as a mathematical term has no factual meaning: the truth or falsehood of a sentence involving only mathematical terms you never check via establishing *empirical* facts. The specific body of mathematics – e. g. calculus or arithmetic – what people employing the given scientific theory use in their computations is considered to be the ***'mathematical apparatus***' of the theory, and its terms are just gears, technical devices, but do not belong to the core of the theory.

And logical empiricists were intolerant of another set of conceptual gears, namely those terms of a given scientific theory that do not denote any directly observable things, but are used to explain – or as some logical empiricists preferred to call it: 'organize' – observable empirical data. (This distinction has a long history. For Newton, the inventor of the classical Theory of Gravity, the notion of 'gravitational force' – as a force exerting action-at-a-distance without the mediation of anything else between the two bodies of matter – was 'inconceivable' and 'an absurdity'.) Such conceptual devices are considered by the logical positivists to be ***theoretical terms*** – which can change with the development of science, and have no valid explanatory power – as contrasted with the theory's genuine terms, that is the ***observational terms***.

Logical positivism may be dead as a doornail, however, notions of 'the mathematical apparatus of a scientific theory', and 'theoretical terms' keep on haunting even the minds of present day philosophers of science.

In my talk, after a brief summary of some important pieces of the relevant literature, I will present 3 metamathematical ideas of how we can give rigorous representations of these persistent philosophical intuitions.

Concept Algebras and Conceptual Distance

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The fundamental objects of study in mathematics are those objects that are equipped with a *structure*. A wide range of mathematicians are fascinated with the beauty of specific mathematical structures. While other mathematicians are more concerned with investigating the ability to have larger structures built from the smaller ones. Another interesting thing is studying the *concepts* that one can define in a structure. This is an important, and for some extent a subtle, aspect of study.

By a *concept*, it is meant a relation (of any finite arity) that can be defined on the structure in hands, using its own language. For example, given a group G, the notion of "the center $Z(G)$ of G" is a concept, it is defined by the formula $\forall y\ (xy = yx)$. But the set of "all elements of finite order in G" is not a concept; the formal rendition, $\exists n\ (x^n = e)$, of this set is not a formula of the language of groups (since the quantifier ranges over the natural numbers, not the elements of the group).

The *concept algebra* of a structure $\mathfrak{A}$ is the algebra that consists of all concepts of $\mathfrak{A}$. These algebras were introduced by A. Tarski around 1947, see [1]. He called them *cylindric algebras*, this name refers to an essential geometric meaning, see Figure 1. These algebras attracted a large number of mathematicians. They found interesting realizations and applications in different disciplines, e.g., Mathematics, Computer science, Linguistics, Philosophy and Logic.

Figure 1: Cylinders in concept algebras

There are many ways in which one can use concept algebras to provide a qualitative and quantitative study of the differences between mathematical structures (and mathematical theories in general). For instance, in [2], we introduced a notion of distance that counts the minimum number of concepts that distinguish two given structures. The idea is simple:

Let K be a class of mathematical structures. The *conceptual network of* K is defined to be the 'infinite' graph whose nodes are the elements of K, and with two types of edges: red edges connecting the structures whose concept algebras are isomorphic, and blue edges connecting any two structures if they cannot be adjacent by a red edge; the concept algebra of one of them is embeddable into the concept algebra of the other one; and we can add one element to the small concept algebra to generate the bigger concept algebra.

Thus, the *conceptual distance* between two structures in K is the minimum number of blue edges among all paths connecting these structures in the network of K. This distance can take the value ∞. We note that the definition in [2] is written in the terminology of mathematical logic, and many interesting theorems in that direction have been obtained.

Calculating the conceptual distance between specific structures may provide quite interesting results, e.g., [2, Theorem 5.1]. In addition to that, investigating the conceptual network of a class is thought-provoking. Here is an example: We say that a conceptual network is *strongly connected (SC)* if the conceptual distance between any two of its nodes is finite. A *SC component* of a conceptual network is a maximal *SC* subnetwork (subnetwork is the analogous terminology of subgraph).

Proposition (Khaled et. al. [2])**.** *The conceptual network of all finite groups has infinitely many SC components. Moreover, any two finite groups of different orders cannot lie in the same SC component.*

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Comparing classical and relativistic dynamics in terms of inelastical collisions

Koen Lefever Gergely Székely

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Abstract

In previous research (Lefever and Székely 2018), we have compared classical kinematics with relativistic kinematics in the framework of (Andréka et al. 2012) by constructing a translation function which uses a Galilean transformation and a Lorenz transformation to translate between classical and relativistic co-ordinates. By using logical interpretations and definitional equivalence we have shown that those theories become equivalent if a primitive ether is added to the relativistic axioms, or in other words: the presence of absence of a priviliged ether frame is sufficient to distinguish the classical kinematics from relativistic kinematics.

We are currently extending our work into dynamics, and at first examine the case of non-elastical collisions. Axioms for non-elastical collisions have already been introduced in (Andréka et al. 2008), (Madarász and Székely 2014), and (Madarász et al. 2014). We introduce a variant of these axioms expressed in the three-sorted first-order logic language $\{ B, IOb, Q\,; Ph, +, \cdot, \leq, W, M\}$, where B is the set of bodies, IOb the set of inertial observers, Q the set of quantities, Ph the set of light signals (photons), $+$, $\cdot$ and $\leq$ the operators on quantities, W the 6-place world-view relation of sort $IOb \times B \times Q^4$ which formulates coordinatization, and M the 3-place mass relation of which the first two arguments are of sort B and the third argument is of sort Q, reading $M(k, b, q)$ as "the mass of body b is q according to body (observer) k."

We report on our ongoing research to construct interpretations of classical collisions as relativistic colissions, and the other way round. we use the translation functions from the previous results connecting kinematics to translate spatio-temporal quantities. Then, we will show what changes to the theories are necessary such that they become definitionally equivalent, pinpointing the exact simmilarities and differences between both theories for dynamics.

Our aim is to provide a deeper understanding of the logical connection between classical and relativistic dynamics.

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Concept algebra of special relativistic spacetime

Judit Madarász

Joint research with H. Andréka, I. Németi, and G. Székely

We explore the first-order logic conceptual structure of special relativistic spacetime: We describe the algebra of concepts (explicitly definable relations) of Minkowski-spacetime, and draw conclusions such as "the concept of lightlike-separability can be defined from that of timelike-separability by using four variables but not by using three variables", or "no non-trivial equivalence relation can be defined in Minkowski-spacetime", or "there are no interpretations between the classical (Newtonian) and the relativistic spacetimes, in either direction".

We also show that while the algebras of zero-ary and unary concepts are trivial, two-element ones, the algebra of binary concepts has 16 elements and the algebra of ternary concepts is infinite. These results are true over arbitrary ordered fields as the structure of quantities. Concerning the algebra of concepts over real-closed fields only, the algebra of ternary concepts is atomic, and we give a concrete mathematical description for it. Similar, but different, results are true for classical spacetime and Euclidean geometry. For example, the algebra of binary concepts of classical spacetime has only 8 elements and that of Euclidean geometry has only 4 elements.

Both Leon Henkin and J. Donald Monk expressed the desirability of these kinds of investigations earlier, the above are the first results of this kind.

Lorentzian Structures on Branching Spacetimes

David O'Connell

For several decades, branching spacetimes have been discussed in both the philosophical and logical literature. The most studied approach is that of Belnap, who introduced a logical theory known as BST92 that appropriately generalises the order-theoretic properties of Minkowski spacetimes to the branching regime. Successors of Belnap (particularly Műller [2]) have since constructed and studied a special class of models of BST92 whose histories are order-isomorphic to some Minkowski spacetime of fixed dimension. We will call such models Minkowksian Branching Spacetimes (hereafter MBSTs).

One can view Belnap's BST92 as a generalisation of branching temporal to include (relativistic) spatial components. However, Belnap also had other motivations. In his seminal text, Belnap writes:

"The aim was to contribute to the problem of uniting relativity with indeterminism in a fully rigorous theory." [1]

Although Belnap and his successors have made quite a contribution to this problem, there are two obvious senses in which BST92 and its MBSTs do not suffice as a full resolution:

(1) the MBSTs of BST92 are order-theoretic objects, thus do not possess the relevant structure to be considered spacetimes, and (2) BST92 can only deal with special-relativistic branching.

In this talk we will remove these two limitations. This is done by developing a theory of adjunction spaces that allows time-oriented Lorentzian manifolds (i.e. spacetimes) to be glued to each other along isometric open submanifolds. It can be shown that the resulting glued structures naturally inherit the spacetime structures of its constituents, effectively creating non-Hausdorff spacetimes that are conveniently well-behaved with regards to their geometric and causal structures.

The mathematical machinery of adjunction spaces allows us to recreate Müller's construction of MBSTs at the level of the Lorentzian structure of Minkowksi spacetime. The result will be a class of MBSTs possessing natural topological, smooth and Lorentzian structures, in such a way that every history is now isometric to a given Minkowski spacetime. This allows us to view MBSTs as (non-Hausdorff) spacetimes in their own right, thereby removing the first of the two limitations listed above.

As for the second limitation, we will reproduce Múller's construction for arbitrary spacetimes, which motivates the definition of a new class of objects called Lorentzian Branching Spacetimes. The main result is that from a given spacetime M, one can construct a class of branching spacetimes whose "histories" are isometric to M. We will show that any LBSTs built from M are causally well-behaved, provided M itself is. Finally (time permitting) we will discuss LBSTs' place within the thorny issue of topology change in GR.

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When the foundations of mathematics meets physics - applying Martin-Löf's ideas

Per Martin-Löf in his [1983], explains that there is a significant difference between the concepts “inference” and “consequence” that was present in the study of logic for most of history, but has been lost in the recent centuries. Martin-Löf [2017] presented a view on mathematical reasoning that follows a formula of speech-acts, with assertions and judgments rather than propositions and conclusions. Combining the ideas he has presented with some results from the debate on formalization in the philosophy of mathematical practice allows for making connections between reality and mathematical statements that are more or less unreachable with the systems that are currently used by logicians working on the foundations of mathematics. This talk will give an overview of Martin-Löf’s discussion on interpreting the basic concepts in logic, and I will argue that adopting his interpretation would allow them to be more easily applied in areas outside of pure mathematics.

Key words: formalization, philosophy of mathematical practice, philosophy of science, applied logic

AXIOMATIC AND GENETIC METHODS OF CONCEPT- AND THEORY-BUILDING: AN ATTEMPT OF SYNTHESIS

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In 1900 David Hilbert distinguishes between the axiomatic method known today after his name and the more traditional *genetic* method of concept- and theory-building in mathematics and science, which involves construction of complex mathematical objects from certain primitive objects [1]. In the introductory part of his 1934 volume co-authored with Paul Bernays Hilbert develops a different perspective on the genetic method and suggests that the axiomatic method in the narrow sense of the word is a part of a more general method of theory-building that Hilbert now calls interchangeably genetic and constructive. According to this mature Hilberts view the constructive method is exemplified in history by Euclids *Elements*, Newtons *Principia* and Clausiuss works in Thermodynamics [2].

Building on Hilberts insight on the constructive axiomatic method I attempt to provide it with a modern formal specification and epistemological foundation. This includes using the Gentzen-style formal syntax along with a proof-theoretic semantics and relaxing the standard rigid distinction between logical and extra-logical semantics of formal theories. More specifically I consider the Homotopy Type theory as a formal tool that helps one to identify the logical part of a given theory internally. Finally I argue that the constructive version of axiomatic method is more apt to represent mathematical and scientific theories than the standard formal axiomatic method. The present paper develops ideas earlier presented in my [3]

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How probabilistic networks can learn scientific concepts

In this paper I will explore the relationship between learning and probabilistic networks and show how learning of different types of concepts can be implemented in dictionary-based networks. I begin by defining a learner in terms of a probabilistic network in which each vertex is a special object called a dictionary and defining the notion of (concept) learnability for dictionary-network learners. Finally, I will argue that learning scientific concepts is much easier because of the possibility of reliable supervised learning.

Learner in a probabilistic network. I will define a learner, understood as a learning function [3], in the context of a probabilistic network in which vertices are special objects called dictionaries. First, I will present the notion of a dictionary together with its natural implementation in terms of a mapping object as used in Python programming language.

The fundamental intuition behind dictionary-based networks is that of a *concept.* Let $\mathcal{D}$ be an infinite data stream comprising in a series $\mathcal{D} = \varepsilon_0, \varepsilon_1, \varepsilon_2, \dots$ where each datum ε_n, $n = 0, 1, \dots$ has the form `given A, B`. Dictionaries are created based on a data stream $\mathcal{D}$. With each new datum ε_n in $\mathcal{D}$, either a key is added to an existing dictionary, or a new dictionary with a key is created. A set of all dictionaries $\mathfrak{d}_i$, where $i = 0, 1, \dots$, will be denoted as $\mathfrak{D}$. Each $\mathfrak{d}_i$ can be represented as a set of ordered pairs of the form $\langle k_i, v_i \rangle$, where k_i is a key in dictionary $\mathfrak{d}_i$ and v is a value chosen from the set of available values $\mathcal{V}$, which can take the form $\mathcal{V} = \{0, 1\}$, or the form of an interval $\mathcal{V} = [0, 1]$.

A semantic network is a knowledge base on which a learner will be able to update using pre-defined rules. Moreover, the fact that the same type of value is used for each key, allows interpreting the numerical values as *links* in the sense of the Semantic Link Network (SLN) scheme [5]. Dictionaries form a semantic network which has a natural respresentation in terms of graphs. In such a graph, two dictionaries are connected by a node if the following condition holds:

Connecting vertices. Two vertices $\mathfrak{d}_1$, $\mathfrak{d}_2$ are connected with a node iff $\mathfrak{d}_1$ occurs as a key in $\mathfrak{d}_2$, or *vice versa.*

I will show how to impose stricter conditions in terms of probabilities for connecting two vertices in order to make use of the values associated with the keys in particular dictionaries. For now, however, it suffices to say that the dictionaries form a network in which non-trivial conditions for connections between vertices is possible, that it, it is not the case that every dictionary is connected to all other dictionaries.

Concept learnability. In general terms, a probabilistic network is a graphical model encoding probabilistic relationships between variables of interest. Besides the numerical parameters of the probability distribution, probabilistic networks accommodate qualitative influences between variables, which originate from prior knowledge about the variables or data [4]. By applying our prior knowledge about scientific concents, updating on dictionaries can be relativized to the particular empirical requirements for each concept.

I will demonstrate how this relative update can be implemented relying on algorithmic theory of meaning. That is, the meaning of each concept will be understood as the "algorithm" for computing the object. For dictionary-based probabilistic network, the algorithm will always yield a conditional probability from $\mathcal{V} = [0, 1]$. A concept $\mathbf{c}$ is considered learnable if the learner will converge on the probabilities in $\mathbf{c}$-dictionary which are within a specially defined acceptable limit.

Scientific and ordinary concepts. For natural language concepts, which are vague or the meaning of which changes according to usage, learning requires epistemic planning on the side of the agent [2, 1]. However, learning in the scientific context often relies on well-defined concepts, which can be given to the learner in the process of supervized learning. This means that scientific concepts can be learned much more reliably and quickly than concepts for which the update on the meaning of a concept is required. In my paper I will show examples of concepts, like set membership, which in a scientific context take a well-defined algorithmic form. I will also show how learning them is easier than learning most concepts used in everyday language.

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Omitting types in finite variable fragments of first order logic

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Fix $2 < n < m \leq \omega$. L_n denotes $L_{\omega,\omega}$ restricted to the first n variables. The notion of an m-square representation (model) of a cylindric algebra of dimension n, briefly a CA_n (L_n theory) is defined in [4]. For $2 < n \leq m < k \leq \omega$, every k square ordinary is m–square, but the converse may be false. Any (ordinary) model M is a limiting case; it is ω-square. We obtain results of the form: *There exists a countable, complete and atomic L_n first order theory T, meaning that the Tarski Lindenbuam quotient algebra $\mathfrak{Fm}_T$ is atomic, such that the type Γ consisting of co-atoms $\mathfrak{Fm}_T$ is realizable in every m–square model of T, but Γ cannot be isolated using $\leq l$ variables, where $n \leq l < m \leq \omega$.* The last statement denoted by $\Psi(l, m)$, short for Vaught's Theorem (VT*) fails at (the parameters) l and m.* Let $\mathsf{VT}(l, m)$ stand for VT *holds at l and m*, so that by definition $\Psi(l, m) \iff \neg\mathsf{VT}(l, m)$. We also include $l = \omega$ in the equation by defining $\mathsf{VT}(\omega, \omega)$ as VT holds for $L_{\omega,\omega}$: Atomic countable first order theories have atomic countable models. The following, where G^m_ω denotes the ω rounded game played on atomic networks of a CA_n using m nodes, is proved in [4].

Lemma 1 . *Let $2 < n < m$. If $\mathfrak{A} \in \mathsf{CA}_n$ is finite and $\forall$ has a winning strategy in $G^m_\omega(\mathsf{At}\mathfrak{A})$, then $\mathfrak{A}$ does not have an m–square representation.*

Theorem 2 . *Let $2 < n < \omega$. Then there exists $\mathfrak{B} \in \mathsf{Cs}_n$ such that its Dedekind-MacNeille completion, namely, $\mathfrak{CmAt}\mathfrak{B}$ does not have an $n+3$-square representation, a fortiori, $\mathfrak{CmAt}\mathfrak{A} \notin \mathbf{S}\mathsf{Nr}_n\mathsf{CA}_{n+3}$.*

Proof. **(a) Blowing up and blurring a finite rainbow algebra:** Take the finite CA_n rainbow algebra $\mathfrak{D}$ where the reds R is the complete irreflexive graph n, and the greens are $\mathsf{G} = \{\mathsf{g}_i : 1 \leq i < n-1\} \cup \{\mathsf{g}^i_0 : 1 \leq i \leq n+1\}$. Denote $\mathfrak{D}$ by $\mathsf{CA}_{n+1,n}$ and denote its finite atom structure by $\mathbf{At}_f$. One then replaces the red colours of the finite rainbow algebra of $\mathsf{CA}_{n+1,n}$ each by infinitely many reds (getting their superscripts from ω), obtaining this way a weakly representable atom structure $\mathbf{At}$. The resulting atom structure after 'splitting the reds', namely, $\mathbf{At}$, is like the weakly but not strongly representable atom structure of the atomic, countable and simple algebra $\mathfrak{A}$ constructed in [3], the sole difference is that we have $n+1$ greens and not infinitely many as is the case in [3]. Let $\mathfrak{B} = \mathfrak{Tm}\mathbf{At}$. We show that the term algebra $\mathfrak{B}$ is as required.
(b) Embedding $\mathsf{CA}_{n+1,n}$ into the complex algebra $\mathfrak{Cm}\mathbf{At}$: To start with, we Identify r with r^0, so that we consider that $\mathbf{At}_f \subseteq \mathbf{At}$. Let CRG_f be the class of coulored graphs on $\mathbf{At}_f$ and CRG be the class of coloured graph on $\mathbf{At}$. By the above identification, we can assume that $\mathsf{CRG}_f \subseteq \mathsf{CRG}$. Write M_a for the atom that is the (equivalence class of the) surjection $a : n \to M$, $M \in \mathsf{CGR}$. Here we identify a with $[a]$; no harm will ensue. We define the (equivalence) relation $\sim$ on $\mathbf{At}$ by $M_b \sim N_a$, $(M, N \in \mathsf{CGR}) \iff M_a(a(i), a(j)) = \mathsf{r}^l \iff N_b(b(i), b(j)) = \mathsf{r}^k$, for some $l, k \in \omega$, and otherwise M_b and M_a

are identical. We say that M_a is a *copy of* N_b if $M_a \sim N_b$ (by symmetry N_b is a copy of M_a.) Now we define the map Θ from $\mathsf{CA}_{n+1,n} = \mathfrak{Cm}\mathbf{At}_f$ to $\mathfrak{Cm}\mathbf{At}$, by $\Theta(X) = \bigcup_{x\in \mathbf{At_f}} \Theta(x)$ ($X \subseteq \mathbf{At}_f$), by specifing first its values on At_f, via $M_a \to \sum_j M_a^{(j)}$; each atom maps to the suprema of its copies. If M_a does not have a red label, then by $\sum_j M_a^{(j)}$, we understand M_a. This map is well-defined because $\mathfrak{Cm}\mathbf{At}$ is complete. Then f is an injective homomorphim. **(c) $\forall$ s winning strategy in** $G_\omega^{n+3}(\mathsf{At}\mathsf{CA}_{n+1,n})$: $\forall$ has a winning strategy in the Ehrenfeucht–Fraïssé forth private game played between $\exists$ and $\forall$ on the complete irreflexive graphs $n+1$ and n, namely, in $\mathsf{EF}_{n+1}^{n+1}(n+1,n)$ (using $n+1$ pebble pairs in $n+1$ rounds). This game lifts to a graph game [p.841, 2] on $\mathbf{At}_f$. Now $\forall$ lifts his winning strategy from the private Ehrenfeucht–Fraïssé forth game, to the graph game on $\mathbf{At}_f = \mathsf{At}(\mathsf{CA}_{n+1,n})$. He bombards $\exists$ with cones having the same base with green tints, demanding that $\exists$ delivers a red label each time for the succesive appexes of the cones he plays. He will need two more nodes, i.e $n+3$ nodes in the graph game to win. By Lemma 1 $\mathsf{CA}_{n+1,n}$ does not have an $n+3$ square representation. hence $\mathfrak{Cm}\mathsf{At}\mathfrak{B}$ also has no $n+3$ square representation, since the former embeds in the latter. □

Corollary 3 . *Let $2 < n \leq l < m \leq \omega$. Then $\Psi(l,m)$ holds for $l = n$ and $m \geq n+3$ and any finite $2 < n < l < \omega$ and $m = \omega$.*

Proof. Let $\mathfrak{B} = \mathfrak{Tm}\mathbf{At}$ constructed in Theorem 1 be an atomic countable Cs_n such that $\mathfrak{Cm}\mathsf{At}\mathfrak{B}$ does not have an $n+3$-square representation. We can and will assume that $\mathfrak{B} = \mathfrak{Fm}_T$ for some countable complete atomic L_n theory T. Let Γ be the type consisting of co-atoms of $\mathfrak{Fm}_T$, that is to say, $\Gamma = \{\phi : \neg\phi_T \in \mathsf{At}\mathfrak{Fm}_T\}$. Then Γ is non-principal because $\mathfrak{Fm}_T$ is atomc. Furthermore, Γ cannot be omitted in an $m+3$-square model, else this gives a complete $m+3$-square representation of $\mathfrak{B}$, which induces an $m+3$-square representation of $\mathfrak{Cm}\mathsf{At}\mathfrak{Fm}_T = \mathfrak{Cm}\mathsf{At}\mathfrak{B}$, contradiction. The second part follows from the construction in [1] where for each $2 < n < l < \omega$, a countable atomic $\mathfrak{A} \in \mathsf{Cs}_n \cap \mathsf{Nr}_n\mathsf{CA}_l$ is constructed such that $\mathfrak{Cm}\mathsf{At}\mathfrak{A} \notin \mathsf{RCA}_n$. Like before, assume that $\mathfrak{A} = \mathfrak{Fm}_T$. Suppose for contradiction that ϕ is an l-witness, so that $T \models \phi \to \alpha$, for all $\alpha \in \Gamma$, where recall that Γ is the set of coatoms. We can assume that $\mathfrak{A}$ is a set algebra with base M say. Let $\mathsf{M} = (M, R_i)_{i\in\omega}$ be the corresponding model (in a relational signature) to this set algebra. Let ϕ^{M} denote the set of all assignments satisfying ϕ in M. We have $\mathsf{M} \models T$ and $\phi^{\mathsf{M}} \in \mathfrak{A}$, because $\mathfrak{A} \in \mathsf{Nr}_n\mathsf{CA}_l$. But $T \models \exists x\phi$, hence $\phi^{\mathsf{M}} \neq 0$, from which it follows that ϕ^{M} must intersect an atom $\alpha \in \mathfrak{A}$. Let ψ be the formula, such that $\psi^{\mathsf{M}} = \alpha$. Then it cannot be the case that $T \models \phi \to \neg\psi$, contradiction.

Theorem 4 . *If there exists a finite relation algebra having a so-called $m-1$ strong blur but no m-dimensional relational basis, then for $2 < n \leq l \leq m \leq \omega$, $\mathsf{VT}(l,m)$ holds $\iff$ $l = m = \omega$.*

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Do you see what I see? Joint observation in Barbourian universes

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Extended abstract

Barbour presented an atemporal world called Platonia in his work[3][4]. Coming to terms with this representation can be challenging to some. However, I attempt to present a formal system based on temporal logics that would provide insight into the workings of such an atemporal world. I introduce a variation of Belnapian branching structures (as the original Branching spacetimes [5] or Branching continuations [6]) that is based on Barbour's Platonia and call it Barbourian branching structures. The aim of this contribution is to extend and complete the preceding work (for example [1]) and present how different observers interact in the Barbourian branching structures and how the use of multiple observers influences the veracity of statements in Barbourian branching structures, especially focusing relativistic scenarios as the Twin paradox[2].

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Why did such serious people take so seriously axioms which now seem so arbitrary?

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"Why did such serious people take so seriously axioms which now seem so arbitrary?" John Stewart Bell complains in his *Speakable and unspeakable*, and continues:

> I suspect that they were misled by the pernicious misuse of the word 'measurement' in contemporary theory. This word very strongly suggests the ascertaining of some preexisting property of some thing, any instrument involved playing a purely passive role. Quantum experiments are just not like that, as we learned especially from Bohr. The results have to be regarded as the joint product of 'system' and 'apparatus,' the complete experimental set-up. But the misuse of the word 'measurement' makes it easy to forget this and then to expect that the 'results of measurements' should obey some simple logic in which the apparatus is not mentioned. The resulting difficulties soon show that any such logic is not ordinary logic. It is my impression that the whole vast subject of 'Quantum Logic' has arisen in this way from the misuse of a word. I am convinced that the word 'measurement' has now been so abused that the field would be significantly advanced by banning its use altogether, in favor for example of the word 'experiment.' (p. 166.)

We do not want to avoid the word 'measurement' in this paper, but we do take into account that the outcome of a measurement is a joint product of the system and the measurement operation. Namely, we describe a typical empirical scenario in the following way: One can perform different measurement operations on a physical system, each of which may have different possible outcomes. The performance of a measuring operation is regarded as a physical event on par with the measurement outcomes. Empirical data are, exclusively, the observed relative frequencies of how many times different measurement operations are performed and how many times different outcome events occur, including the joint performances of two or more measurements and the conjunctions of their outcomes.

It can be easily shown that the empirical data always admit a classical probabilistic description in a suitable Kolmogorovian probability space, no matter whether the observed phenomena in question belong to classical or quantum physics (in accordance with the Kolmogorovian Censorship Hypothesis). The probability measure however essentially depends on the frequencies with which the measurement operations are performed; that is, on circumstances outside of the physical system under consideration; for example, on the free choice of a human.

Under some plausible—and empirically easily verifiable—assumptions about the joint measurements, we can isolate a notion which is independent from the external circumstances and can be identified with the system's own state, in the sense that it characterizes the system's probabilistic behavior against all possible measurement operations.

In the second part of the talk I present a representation theorem, according to which *everything that can be described in empirical/operational terms*, if we find it convenient, can be represented in the Hilbert space quantum mechanical formalism. There always exists:

(1) a suitable Hilbert space, such that

(2) the outcomes of each measurement can be represented by a system of pairwise orthogonal closed subspaces,

(3) the states of the system can be represented by suitable density operators, and

(4) the probabilities of the measurement outcomes can be reproduced by the usual trace formula of quantum mechanics.

(5) A measurement yields a given outcome with probability 1 if and only if the state of the system is a pure state with state vector contained in the subspace representing the outcome event in question (or a mixed state obtained from such pure states by convex combinations).

Moreover, if appropriate, one can label the possible outcomes of a measurement with numbers, and to talk about them as the measured values of a physical quantity. Each such quantity

(6) can be associated with a suitable self-adjoint operator, such that

(7) the expectation value of the quantity, in all states of the system, can be reproduced by the usual trace formula applied to the associated self-adjoint operator,

(8) each measurement result is equal to one of the eigenvalues of the operator, and

(9) the corresponding outcome event is represented by the eigenspace belonging to the eigenvalue in question.

The theorem might suggest that the basic axioms of quantum theory simply follow from the fact that the system can be described in empirical/operational terms. This is almost true, but the QM-like representation satisfying (1)–(9) is not completely identical with the standard QM. Several of the standard QM claims are missing; for example, no connection between commutation and joint mensurability, no one-to-one correspondence between operational physical quantities and self-adjoint operators, not all state vectors/density operators represent a physical state of the system, etc. However, interestingly, the missing elements of the standard quantum mechanical claims are exactly those that have been often questioned in the past decades. Another interesting consequence of our representation theorem is that, contrary to the long-standing debates about the quantum-logical connectives, the lattice-theoretic meets do have suitable empirical meaning regardless whether the corresponding projectors commute or not, while the lattice-theoretic joints, in general, have nothing to do with the disjunctions of the outcome events.

Now, can or cannot classical kinematics interpret special relativity?

Gergely Székely

This talk is based on joint work with Hajnal Andréka, Koen Lefever, Judit X. Madarász, and István Németi.

With Koen Lefever, we have recently constructed a logical interpretation from an axiomatic theory of special relativity to that of classical kinematics, see [1] and [2]. In more detail: We took an axiom system of special relativity developed by the Andréka–Németi group and we developed an axiomatic theory of classical kinematics using the same first-order logic language. That the used axioms systems capture what they intend to capture is justified by the facts that the transformations between inertial observers are Poincaré and Galilean transformations, respectively. Then in classical kinematics, we were using Einstein's construction of simultaneity using light signals and a natural synchronization of clocks to construct relativistic coordinate systems for classical observers. We have shown that this gives a logical interpretation of special relativity into classical kinematics because these new coordinate systems of classical observers satisfy all the axioms of special relativity.

Partly motivated by the above research, with Hajnal Andréka, Judit X. Madarász, and István Németi, we have started to investigate the concept algebras of some natural "standard" of models special relativity and classical kinematics over the field of real numbers. We have learned several interesting things. Among others, we have shown that there are no interpretations between these "standard" models of classical and relativistic spacetimes, in either direction, see [3] and [4].

In this talk, we will recall these two results and shed light on why there is no logical contradiction here even though these results intuitively state just the opposite. Spoiler alert: "the devil hides in the details" as always.

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Predicate logic with explicit substitution

Richard J. Thompson

The main tool of the Budapest Logic and Relativity group is first order logic (FOL for short), cf. e.g., [1]. This talk is about an important feature of FOL: we can substitute one variable for another one in a formula. This feature is important because it can reflect that variables are only "placeholders" in formulas: where they are situated in a formula is what counts, and not their "names".

In classical logic, Tarski's definition of the satisfaction of a formula by a given assignment can be extended, in a straightforward way, to define the intended meaning of the formula B that we obtain by substituting the variable v_i for v_j in a formula A. Further, Tarski showed in [6] that this B has the same meaning as $\exists v_j(v_j = v_i \wedge A)$. That is, substitution of variables can be expressed/handled in an explicit way in the presence of the identity.

However, there are logical systems where we do not have equality in our basic set of logical connectives. Not to go too far, classical FOL without equality is such. Another example when we do not have equality is intuitionistic logic. In these cases Tarski's way of dealing with substitution does not work (makes no sense). The problem of treating substitution in logics where one does not have equality was raised several places, for example in [3, p.116]. Charles Pinter (and others) suggested (in the 1970's) to handle substitution in an explicit way, namely we can take substitutions to be new basic logical connectives (next to the Boolean connectives and quantifiers). Cf. e.g., [5]. For example, taking two kinds of basic substitutions $S(i/j)$ and $S(i,j)$ with appropriate logical axioms does the job (the intended meanings of $S(i/j)$ and $S(i,j)$ are, respectively, substituting v_i by v_j , and permuting v_i and v_j).

In this talk, I consider predicate logic (also sentential logic) with new logical connectives $S(i/j)$, the replacement of the variable v_i by v_j, and $S(i,j)$, the transposition of v_i and v_j. I supply proof systems that are complete with respect to these new formulas: a new formula is provable if and only if it is valid under the intended meaning as given by Tarski.

In the proof systems, we have axioms which express how we can obtain a standard (or normal) form in which all the substitution operations $S(i/j)$ and $S(i, j)$ have been driven inside and appear only before other such operations and atomic formulas. These latter can be identified with plain atomic formulas, and this proves that by adding explicit substitutions we get a conservative extension of intuitionistic logic.

The explicit use of substitution operators enables us, when dealing with a classical sequent calculus having an invertibility preserving formulation (as in [2] or [4]) to proceed in slight local steps, avoiding the disruption of alphabetic variance. At the meta-logical level it becomes possible to make such simplifications as closing out axiom schemes by applying, not arbitrary universal quantifications, but only quantifications of variables free in the schema. (This requires that when a finite permutation is made of the substitutional variables in an axiom, then the result is also an axiom – all substitutional variables are alike.)

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New data on space curvature may support non-inflationary geometrical solution for the horizon problem

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Abstract

Dark matter is not detected directly, while dark energy is not understood theoretically on a satisfactory level. However, the concordance (ΛCDM) cosmological model accompanied by an appropriate inflationary scenario describes the Universe very well. The inflation model provides explanations for the monopole, flatness, and horizon problems. The model is supported by the experimental data, providing a natural mechanism for the observed nearly scale invariant spectrum of primordial adiabatic density fluctuations, with an amplitude consistent with generating the observed large scale structure of the Universe. It is also in agreement with the absence of certain correlations in the cosmic microwave background (CMB), the non-Gaussianities. The Gaussian nature of perturbations, as inferred from CMB temperature anisotropies, is due to the Gaussian statistics in the case of a single quantum field. However, inflation is not consistent with the observed large scale angular correlations in the CMB data. The models require angular correlation at all angles, not only at angles up to ~ 60^0, because inflation occurred at all scales.

In addition, the recent Planck data impose strict constraints on the shape of the inflation potential disfavoring the simplest inflation models. Also, quantum fluctuations that produce random variations of the inflationary energy, similarly produce random warps in space that propagate as waves of spatial distortion across the Universe once inflation ends. These cosmological gravitational waves are not detected. This distortion also contributes to the hot and cold spots in the CMB radiation and causes light to have a certain preferred orientation for its electric field, depending on whether the light comes from a hot or cold spot, but that polarization is also not detected.

Another unanswered problem is that after the time when inflation ended to the time when CMB is produced (the period of 380,000 years), the temperature of the universe changed from 10^{29} K to 3000 K, and the densities changed from 10^{38} kg/m^3 to 10^{-17} kg/m^3. On another side, the high degree of isotropy of the CMB temperature on the level of 0.01% requires that the density variations from one region of space to another at the time when CMB is emitted must have been smaller than a few parts in 10^{-5}. This means that the changes of the density in any part of the universe were the same as to the 60 orders of magnitude, which is statistically unlikely. The problem is that after inflation ended, some parts of the Universe are not anymore causally connected, and there is no reason that they will have the same density and the same temperature at the time of decoupling. It is also hard to explain this uniformity theoretically. For instance, it is not clear how at the end of inflation, during reheating, the energy of vacuum was transferred to ordinary matter and radiation, which particles are created, and how latter some particles effectively stopped interacting with the rest of matter and radiation and become cold dark matter. These uncertainties do not allow us to make predictions that will be accurate on 60 decimal places, which is needed to explain observed uniformity in the CMB.

This is similar to the horizon problem, but after inflation, inflation does not help to solve it. So, how do we explain the CMB uniformity? Or is it possible that the uniformity of the CMB does not mean the uniformity of the space? Is it possible that we are always measuring the CMB

coming from the same point and not from the different parts of the Universe, which will explain its uniformity?

The recent data collected by the Planck satellite suggests that the Universe is actually curved and closed [1], like an inflating sphere. As the authors of [1] demonstrate, positive spatial curvature can explain the anomalous (enhanced) lensing amplitude in the CMB power spectra and remove the Planck data tension with respect to the cosmological parameters derived at different angular scales.

Encouraged by these findings, we present our new results on the alternative interpretation of the CMB data [2]. We argue that the observed CMB uniformity does not mean that space was uniform at the time of decoupling. A large-scale homogeneity and isotropy are not required by the classical theory of general relativity. It is well known that in the Big Bang models, the homogeneity of space cannot be explained, being assumed in initial conditions. We demonstrate that within a simple extension of the ΛCDM model, in the case of the positive curvature Universe, there is an elegant solution of the horizon problem without inflation. Under the proper parameter choice, light travels between the antipodal points during the age of the Universe. Thus, one can suggest that the observed CMB radiation originates from a very limited spatial region in the vicinity of the antipodal point. Therefore, measuring the same CMB by looking in the opposite directions of the Universe does not reflect the uniformity at the time of decoupling, because we always measure CMB radiation originating from approximately the same antipodal point regardless of the direction of observation.

Small variations in the CMB are possible and observed, but these variations are the result of measuring CMB from a small region and not exactly from a single point, as well as of the interaction between matter and light during its travel. For instance, depending on the direction we choose to measure CMB, light travels through different galaxies and interacts with different amounts of matter. This results in small observed variations in the CMB at large angular scales (as photons pass through large scale structures) by the integrated Sachs – Wolfe effect.

Consequently, the CMB radiation uniformity can be explained without the inflationary scenario. Also, this removes any constraints on the uniformity of the Universe at the early stage and opens a possibility that the Universe was not uniform and that the creation of galaxies and large structures may be caused by the inhomogeneities that originated in the Big Bang. We reach the agreement with the supernovae data and show that changing the amplitude of the initial power spectrum, one can adjust the proposed cosmological model to the CMB anisotropy and that the discussed change is inside the experimentally allowed constrains.

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Where Does General Relativity Break Down?

James Weatherall

One motivation for pursuing a quantum theory of gravity is that general relativity "breaks down" under certain conditions, in the sense of exhibiting pathological or singular behavior. Such behavior, it is sometimes argued, is a signal that a new theory is necessary to describe physics in those regimes; such a theory is then expected to "resolve" the pathological behaviors of general relativity. In this talk, I will consider what sorts of pathologies a quantum theory of gravity might be expected to resolve. I will argue that singularities associated with the divergence of a curvature scalar are the most natural candidates for resolution by a theory of quantum gravity—but, as has long been known in the foundations of general relativity literature, these are not the only pathological features allowed by general relativity. I will then relate this discussion to the so-called strong cosmic censorship conjecture, which states that the maximal Cauchy evolution of generic initial data is inextendible. I will argue that from the perspective of the breakdown of general relativity, the most compelling interpretation of this conjecture is as a way of linking the emergence of Cauchy horizons with curvature singularities, which might then be expected to be resolved by quantum gravity. Recent work on this conjecture by Dafermos and Luk has been taken as evidence that the conjecture may be false in its strongest (C0) form; I will argue that, to the contrary, Dafermos and Luk have provided evidence that the physically relevant form of the conjecture is true, insofar as they show that curvature singularities form at the Cauchy horizon of a perturbed Kerr black hole.