On the relative asymptotic expressivity of probabilistic inference frameworks: an approach via finite model theory

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Abstract. Let L be a first-order language with finite relational signature and let W_n be the set of all L-structures with domain of which is the set of integers from 1 to n. Here, an asymptotic probability distribution is a sequence $P = (P_n : n = 1, 2, ...)$ of probability distributions P_n on W_n. In this talk an inference framework F is a class of pairs (P, L) where P is an asymptotic probability distribution and L is a logic. The inference frameworks that we consider will contain pairs (P, L) where P is determined by a so-called probabilistic graphical model, a concept used in AI and machine learning, and L is a (possibly many valued) logic with capabilities of expressing statements about, for example, (conditional) probabilities or (arithmetic or geometric) averages. We define a notion that an inference framework is "asymptotically at least as expressive" as another inference framework. This relation is a preorder and we describe a (strict) partial order on the equivalence classes of some inference frameworks. The results have bearing on issues concerning efficient learning and probabilistic inference in AI and machine learning, but are also new instances of results in finite model theory about "almost sure elimination" of extra syntactic features (e.g quantifiers) beyond the connectives. Often such a result has a logical convergence law as a corollary.