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Generalized model theory and continuous logic

Abstract. For a countable language L all structures on $\mathbb{N}$ form a Polish S_∞-space under certain topology. Becker has noticed that some basic model theoretic concepts and theorems can be formulated in topological terms as statements about Polish G-spaces. Generalized model theory is an approach which studies G-spaces from this point of view.

In 2017 we together with Barbara Majcher-Iwanow extended the concept of nice topologies of Becker to the general case of Polish G-spaces (Becker assumed that G is a subgroup of S_∞). Let $(\mathcal{Y}, d)$ be a Polish space and $\mathsf{Iso}(\mathcal{Y}, d)$ be the corresponding isometry group endowed with the pointwise convergence topology. Then $\mathsf{Iso}(\mathcal{Y}, d)$ is a Polish group. For any countable continuous signature L the set $\mathcal{Y}_L$ of all continuous metric L-structures on $(\mathcal{Y}, d)$ can be viewed as a Polish $\mathsf{Iso}(\mathcal{Y}, d)$-space. We call this action logic. Note that for any tuple y from $\mathcal{Y}$ the map taking $g \in G$ to $d(y, g(y))$ can be considered as a grey subgroup of G. For any continuous sentence ϕ we have a grey subset of $\mathcal{Y}_L$ taking $M \in \mathcal{Y}_L$ to ϕ^M.

Typical notions naturally arising for logic actions can be applied in the general case of a Polish G-space $\mathcal{X}$ with G as above. If we consider G together with a family of grey subgroups as above, then distinguishing an appropriate family $\mathcal{B}$ of grey subsets of $\mathcal{X}$ we arrive at the situation very similar to the logic space $\mathcal{Y}_L$. These ideas are applied in a framework of generalized model theory towards analysis of Borel/algorithmic complexities of subsets of $\mathcal{Y}_L^k \times \mathsf{Iso}(\mathcal{Y}_L)^l$ corresponding to model theoretic concepts.