

Sum(m)it280  
Booklet of abstracts

HUN-REN Alfréd Rényi Institute of Mathematics



# Invited Talks

## The combinatorics of distance problems

Noga Alon  
Princeton and Tel Aviv

8 Jul  
10:00am  
Gólyavár (main)

All four birthday boys discovered beautiful connections between combinatorial and geometric results. After a very brief discussion of a (very biased) selection of some of these, I will describe a recent joint work with Matija Bucic and Lisa Sauermann about extremal problems for typical norms, mentioning its connection to questions and results of these four amazing researchers.

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## Extremal problems in the hypercube

Maria Axenovich  
Karlsruhe Institute of Technology, Germany

10 Jul  
2:00pm  
Gólyavár (main)

For two (hyper)graphs  $G$  and  $H$ , the extremal number  $ex(G, H)$  is the largest number of edges in an  $H$ -free subgraph of the ground graph  $G$ . Determining  $ex(G, H)$  remains a challenge in general, even when  $G$  is a complete graph  $K_n$ . However, in this case we know exactly what (hyper)graphs  $H$  have a positive or zero Turán density  $\pi(H)$ , where  $\pi(H) = \lim_{n \rightarrow \infty} ex(K_n, H) / \binom{n}{k}$ .

When the ground graph  $G$  is the hypercube  $Q_n$  of dimension  $n$ , we don't even have such a characterisation. In this talk, I will present what we know about  $ex(Q_n, H)$  and how this extremal number relates to the classical extremal numbers of hypergraphs.

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## Strings and Drawing

Jacob Fox  
Stanford University

12 Jul  
2:00pm  
Gólyavár (main)

The study of extremal problems for topological graphs and the study of intersection patterns of curves has a rich history. In this talk, I will highlight recent progress and longstanding open problems. It will be clear that János Pach's influence on the field is pervasive.

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10 Jul  
10:00am  
Gólyavár (main)

## Saturation saturated

Balázs Keszegh

HUN-REN Alfréd Rényi Institute of Mathematics

Extremal combinatorics mostly deals with the maximum size structure that has some property (e.g., the maximum number of edges in a graph avoiding a triangle). Saturation counterparts of these problems were studied earlier for graphs and more recently also for many other combinatorial structures. In these we are looking for the minimum size structure that saturates some property, i.e., one that cannot be extended (e.g., the minimum number of edges in a graph avoiding a triangle in which adding any new edge creates a triangle). We survey results of this type, starting with graphs. Extending the unordered case we consider graphs with an order on their vertices, be it linear, cyclic or bipartite linear (a problem equivalent to forbidden 0-1 matrices) and most recently with an order on their edges. Then we survey results related to saturation problems about forbidden subposets in the Boolean poset. We mostly concentrate on the dichotomy phenomenon that is prevalent in these problems: the saturation function is either bounded (not depending on the size of the input) or it is a big function of the input (say, at least linear), further, we try to characterize these two classes. Finally, we show saturation counterparts of several Ramsey-type problems of graphs, posets and point sets, including the problem of Erdős and Szekeres about finding a large convex subset of points in a given set of points.

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11 Jul  
2:00pm  
Gólyavár (main)

## Problems on Extremal Combinatorics

Younjin Kim

Pohang University of Science and Technology

Extremal combinatorics aims to determine or estimate the maximum or minimum possible cardinality of a collection of finite objects (such as sets, graphs, numbers, vectors, etc.) that satisfy certain requirements. I am particularly interested in Turán-type problems for hypergraphs and graphs. In this talk, I will introduce Erdős-Shelah Conjecture(1972), which I have worked on in collaboration with other coauthors, including significant contributions made with Professor Zoltán Füredi. Additionally, I will discuss Alon-Babai-Suzuki Conjecture(1991), Erdős-Sós Conjecture(1979), and Erdős Nested Cycle Conjecture(1976).

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## Modifying old ideas to get new results on cycles in hypergraphs

Alexandr Kostochka

University of Illinois, Urbana-Champaign

10 Jul  
9:00am  
Gólyavár (main)

We use an idea of Dirac from 1952 to derive exact degree conditions for the existence of hamiltonian Berge cycles in uniform hypergraphs. We modify another idea of Dirac from 1952 to find exact conditions for the same problem when the hypergraphs are 2-connected. We also modify the Hopping Lemma by Woodall from 1973 to find exact degree conditions for the existence of a spanning jellyfish in a 2-connected graph.

The talk is based on joint work with J. Kim, R. Luo and G. McCourt.

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## Intersections of interest

Andrey Kupavskii

Moscow Institute of Physics and Technology

9 Jul  
2:00am  
Gólyavár (main)

Working with Péter Frankl and János Pach in many ways have defined my research path. I wanted to discuss several topics from the joint research with Péter Frankl on intersections and matchings in extremal set theory. I will also cover some recent developments in the field coming from Boolean analysis and spread approximations, and their connections to the work of Péter Frankl and Zoltan Füredi from the 70s and 80s.

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## Submodular functions and limits of matroids

László Lovász

HUN-REN Alfréd Rényi Institute of Mathematics

8 Jul  
2:00pm  
Gólyavár (main)

Limit theories of graphs were started in the early 2000's, and analogous theories have been developed for posets, permutations, and other combinatorial structures. While trying to develop a limit theory for matroids, we have run across an unexpected connection with analysis, namely potential theory, through the work of Choquet in the 1950's. Our work is still in progress, but I can report on some interesting connections and cross-fertilizations between the combinatorial and analytic theories.

This is joint work with Kristóf Bérczi, Márton Borbényi, Boglárka Gehér, András Imolay, Balázs Maga, László Tóth and Dávid Schwarz.

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## New methods for expanders and their applications

10 Jul  
3:15pm  
Gólyavár (main)

Abhishek Methuku  
ETH Zürich

In this talk we will present new methods and tools for expanders and discuss how they can be used to make progress towards several longstanding open problems.

Based on joint works with Bradac, Chakraborti, Janzer, Letzter, Montgomery and Sudakov.

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## Crossing Lemma for multigraphs

12 Jul  
9:00am  
Gólyavár (main)

Géza Tóth  
HUN-REN Alfréd Rényi Institute of Mathematics

Let  $G$  be a simple graph with  $n$  vertices and  $e > 4n$  edges. According to the Crossing Lemma, the number of crossings in any drawing of  $G$  is at least  $c \frac{e^3}{n^2}$ , for a  $c > 0$ . This bound cannot be improved apart from the value of  $c$ . There is no such statement for multigraphs in general. We investigate under what conditions does the statement of the Crossing Lemma, or a similar statement holds for multigraphs.

In particular, we show that if the “lens” enclosed by every pair of parallel edges contains at least one vertex and adjacent edges do not cross, then the original statement holds. A similar, but weaker bound holds if we only assume that no two edges are homotopic, that is, no two parallel edges can be continuously transformed into each other without passing through a vertex.

Joint work with M. Kaufmann, J. Pach, G. Tardos, T. Ueckerdt.

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## Recent progress in Ramsey Theory

9 Jul  
9:00am  
Gólyavár (main)

Jacques Verstraete  
University of California, San Diego

The organizing principle of Ramsey theory is that in large mathematical structures, there are relatively large substructures which are homogeneous. This is quantified in combinatorics by the notion of Ramsey numbers  $r(s, t)$ , which denote the minimum  $N$  such that in any red-blue coloring of the edges of the complete graph on  $N$  vertices, there exists a red complete graph on  $s$  vertices or a blue complete graph on  $t$  vertices.

While the study of Ramsey numbers goes back almost one hundred years, to early papers of Ramsey and Erdős and Szekeres, the long-standing conjecture of Erdős that  $r(s, t)$  has order of magnitude close to  $t^{s-1}$  as  $t \rightarrow \infty$  remains open in general. A recent breakthrough by Campos, Griffiths, Morris, and Sahasrabudhe gives an exponential improvement to the diagonal Ramsey numbers. We focus on off-diagonal Ramsey numbers. It took roughly sixty years before the order of magnitude of  $r(3, t)$  was determined by Jeong Han Kim, who showed  $r(3, t)$  has order of magnitude

$t^2/(\log t)$  as  $t \rightarrow \infty$ . In this talk, we discuss a variety of new techniques, including the modern method of containers, which lead to a proof of the conjecture of Erdős that  $r(4, t)$  is of order close to  $t^3$ .

One of the salient philosophies in our approach is that good Ramsey graphs hide inside pseudorandom graphs, and the long-standing emphasis of tackling Ramsey theory from the point of view of purely random graphs is superseded by pseudorandom graphs. Via these methods, we also come close to determining the well-studied related quantities known as Erdős-Rogers functions and discuss related hypergraph coloring problems.

Joint work in part with Sam Mattheus, Dhruv Mubayi and David Conlon.

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## The modified shifting method and intersecting families with covering number three

Jian Wang

Taiyuan University of Technology.

11 Jul  
9:30am  
Gólyavár (main)

The shifting method, invented by Erdős, Ko and Rado, is a powerful tool in extremal set theory. Many nice properties are maintained by the shifting operator, such as, intersecting, cross-intersecting and matching number at most  $s$ , et al... However, if there is some property that might be destroyed by shifting, then the shifting method often can not be used. Recently, Frankl proposed a modified version of the shifting method called shifted ad extremis, which extends the power of the shifting method. By applying this method, we reprove some classical results with better bounds on  $n$ . In particular we prove that for  $k \geq 7$ ,  $n \geq 2k$ , any intersecting  $k$ -graph  $\mathcal{F}$  with covering number at least three, satisfies  $|\mathcal{F}| \leq \binom{n-1}{k-1} - \binom{n-k}{k-1} - \binom{n-k-1}{k-1} + \binom{n-2k}{k-1} + \binom{n-k-2}{k-3} + 3$ , the best possible upper bound which was proved by Frankl in 1978 subject to exponential constraints  $n > n_0(k)$ .

Joint work with Péter Frankl.

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# Contributed Talks

## Some elementary algebraic and combinatorial methods in the study of zero-sum theorems

Sukumar Das Adhikari

School of Mathematical Sciences, Ramakrishna Mission Vivekananda Educational and Research Institute, Belur Math, Howrah - 711202, WB, INDIA

8 Jul  
11:50pm  
Section 3

Consider a finite abelian group  $G$  (written additively). A sequence  $S = g_1 \cdots g_l$  over  $G$  is called a zero-sum sequence if  $g_1 + \cdots + g_l = 0$ , where  $0$  is the identity element of the group. Inspired by a well known result of Erdős, Ginzburg and Ziv, the area of zero-sum theorems in combinatorial number theory has seen a rapid growth in the recent years.

The area of zero-sum theorems has many interesting results and several unanswered questions. Several authors have introduced interesting elementary algebraic techniques to deal with these problems. We describe some experiments with these elementary algebraic methods and some combinatorial ones, in a weighted generalization in the area of Zero-sum Combinatorics.

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## Circle geometries: Intersecting families and association schemes

Sam Adriaensen

Vrije Universiteit Brussel

11 Jul  
11:15pm  
Section 3

*Circle geometries* (Möbius, Laguerre, and Minkowski planes) capture the geometry of points and non-singular plane sections (called *circles*) of quadratic surfaces in 3-dimensional space. A family of circles in a circle geometry is called *t-intersecting* if any pair of circles of the family shares at least  $t$  points. Call a *t-intersecting* family  $\mathcal{F}$  *trivial* if there are  $t$  points lying on all circles of  $\mathcal{F}$ .

**Theorem 0.0.1** *If  $\mathcal{F}$  is a 1-intersecting family in an ovoidal circle geometry of order  $q$  with  $|\mathcal{F}| > \frac{1}{\sqrt{2}}q^2 + \mathcal{O}(q)$ , then  $\mathcal{F}$  is (essentially) trivial.*

There is no reason to believe that this bound is sharp. Hilton-Milner type families, which are typically the largest non-trivial 1-intersecting families, have size  $\frac{1}{2}q^2 + \mathcal{O}(q)$ . We will discuss why it might be difficult to classify the largest non-trivial 1-intersecting families, as there are other examples whose size is also  $\frac{1}{2}q^2 + \mathcal{O}(q)$ , but which have a vastly different structure.

We will also discuss the known results concerning 2-intersecting families. There are constructions of 2-intersecting families whose size is  $\frac{3}{2}q + \mathcal{O}(1)$  (which numerical evidence suggests is close to the maximum size), but the best known upper bounds are of the form  $\frac{1}{2}q^2 + \mathcal{O}(q)$ .

Lastly, in order to study the classical circle geometries, we are led to association schemes defined on the anisotropic points of embedded polar spaces. While most of these schemes were treated decades ago, the last missing case (elliptic and hyperbolic quadrics in odd characteristic) was only treated recently in general dimension.

Joint work with Maarten De Boeck.

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## Axiomatic geometry of Hilbert through the lens of a combinatorist

11 Jul  
11:40am  
Section 3

Kristina Ago  
University of Novi Sad

In this presentation we are interested in finite incidence structures that satisfy the axioms of the first group of Hilbert's axiom system. We introduce various different classes of finite models. By a finite model with  $n$  points we mean a triple  $(P, L, Pl)$ , where the set of points  $P$  is the set  $[n]$ , while the set of the lines  $L$  and the set of the planes  $Pl$  are some subsets of  $2^{[n]}$  such that the considered axioms are satisfied. Our models mostly arise from well-known combinatorial structures such as finite projective and affine geometries, combinatorial designs, sometimes subject to specific modifications. For each class, we calculate the number of distinct models in that class (with respect to the number of points). Furthermore, we present some results on models that satisfy some additional constraints (e.g., we give a complete list of all the models where each line is incident with exactly two points; there are exactly three such models, with 4, 8 and 22 points). Finally, for smaller values of  $n$ , we determine the exact number of models with  $n$  points. To achieve this, we first reframe the problem into a language that is prone to an attack by a computer. It turns out that the theory of matroids is very helpful for such enumeration up to  $n = 12$ , while for larger  $n$ 's we discuss some other lines of action.

This is a joint work with B. Basic, and partly a joint work with M. Maksimovic and M. Sobot.

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## IDENTIFICATION OF $k$ -DISTANCE MONOTONE BOOLEAN FUNCTIONS

9 Jul  
11:40am  
Section 4

Levon Aslanyan  
Institute for Informatics and Automation Problems of NAS RA Yerevan, Armenia

In this paper, we introduce a class of Boolean functions, called  $k$ -distance monotone Boolean functions, and study query-based identification problem for the functions of this class.

**Definition 0.0.2** : Boolean function  $f : B^n \rightarrow \{0, 1\}$  is called  $k$ -distance monotone if for every two vertices  $\alpha, \beta \in B^n$ , if  $\alpha < \beta$  and Hamming distance between  $\alpha$  and  $\beta$  is at least  $k$ , then  $f(\alpha) \leq f(\beta)$ . For  $k = 1$ , we get ordinary monotone Boolean functions.

In the monotone Boolean function recognition problem based on membership queries, the goal is to determine an unknown function of  $n$  variables using as few queries as possible. The function can be fully recognized by finding all its upper zeros (and/or lower units). The Shannon complexity of finding all upper zeros (lower units) of an arbitrary monotone Boolean function of  $n$  variables is  $C_n^{\lfloor n/2 \rfloor} + C_n^{\lfloor n/2 \rfloor + 1}$ .

Let  $\phi_k(n)$  denote the minimum number of queries for the best algorithm of identification of arbitrary  $k$ -distance monotone Boolean function. We prove that  $\phi_2(n)$  is the sum of the four largest binomial coefficients. Similar formulas can be obtained for  $k \geq 2$ .

Joint work with Gyula O.H. Katona and Hasmik Sahakyan.

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## Vertex-disjoint cycles of different lengths in tournaments

Yandong Bai

Northwestern Polytechnical University, Xi'an, P.R. China

12 Jul  
12:05pm  
Section 3

Cycles are amongst the most fundamental graph objects and have been the focus of extensive study in graph theory. The class of tournaments is an important class of directed graphs. In this talk, we introduce some long-standing conjectures on disjoint cycles in directed graphs, together with our recent results concerning on disjoint cycles of different lengths in tournaments.

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## Ordered Ramsey numbers: recent progress

Martin Balko

Charles University, Prague

8 Jul  
11:25am  
Section 3

An ordered graph is a graph equipped with a linear ordering of its vertex set. The ordered Ramsey number  $R_{<}(G, H)$  of two ordered graphs  $G$  and  $H$  is the minimum positive integer  $N$  such that every red-blue coloring of the edges of the ordered complete graph on  $N$  vertices contains either a red copy of  $G$  or a blue copy of  $H$  as an ordered subgraph.

Despite numerous applications in extremal combinatorics and discrete geometry, the ordered Ramsey numbers were considered relatively recently. Actually, one of the first events where this concept was introduced was at the Sum(m)it:240 conference. Since then ordered Ramsey numbers attracted significant attention and became an active part of graph Ramsey theory.

In this talk, we survey some of the newer results about ordered Ramsey numbers of graphs. In particular, we will focus on one of the well-known problems in the area about estimating on off-diagonal ordered Ramsey numbers of ordered matchings

versus a triangle, which was stated by Conlon, Fox, Lee, and Sudakov, and we discuss multi-color ordered Ramsey numbers. This is based on joint works with Marian Poljak and with Jelínek and Valtr.

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8 Jul  
12:15pm  
Section 1

## Small codes and set-coloring Ramsey numbers

Igor Balla  
Masaryk University

Determining the maximum number of unit vectors in  $R^r$  with no pairwise inner product exceeding  $\alpha$  is a fundamental problem in geometry and coding theory. In 1955, Rankin resolved this problem for all  $\alpha \leq 0$  and in this talk, we will show that the maximum is  $(2 + o(1))r$  for all  $0 \leq \alpha \ll r^{-2/3}$ , answering a question of Bukh and Cox. Moreover, the exponent  $-2/3$  is best possible. As a consequence, we obtain an upper bound on the size of a  $q$ -ary code with block length  $r$  and distance  $(1 - 1/q)r - o(r^{1/3})$ , which is tight up to the multiplicative factor  $2(1 - 1/q) + o(1)$  for any prime power  $q$  and infinitely many  $r$ . When  $q = 2$ , this resolves a conjecture of Tietäväinen from 1980 in a strong form and the exponent  $1/3$  is best possible. Finally, using a recently discovered connection to  $q$ -ary codes, we obtain an analogous result for set-coloring Ramsey numbers.

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8 Jul  
16:15pm  
Section 1

## Some recent results on the Heesch number in two-dimensional and more-dimensional spaces

Bojan Bašić  
University of Novi Sad

The so-called *Heesch number* of a given figure  $T$  is either a nonnegative integer or  $\infty$  that says how many times  $T$  can be completely surrounded by its congruent copies. The Heesch number of  $T$  is  $\infty$  if and only if  $T$  tessellates the plane. The notion of the Heesch number is more interesting for figures that do not tessellate the plane; in that case, the Heesch number can be perceived as a kind of measure how “far” toward a tiling we can advance with congruent copies of the given figure (the larger it is, the figure “behaves more nicely”). It is an open question, called Heesch’s problem, whether the set of nonnegative integers that can be the Heesch number of some figure is bounded from above (in other words, whether there exists the largest possible finite Heesch number).

For the last two decades, the “record-holder” was a figure with Heesch number 5. In the first half of the talk we present a recent construction that produces a figure with Heesch number 6, thus finally breaking this record.

Then we move on to tessellations in more-dimensional spaces, and solve the following asymptotical version of Heesch’s problem: if we let  $d \rightarrow \infty$ , then there is no uniform upper bound on the set of all possible finite Heesch numbers in the  $d$ -dimensional Euclidean space  $\mathbb{E}^d$ . In particular, we show that, for any  $d$ ,  $d > 2$ , there exists a hypersolid in  $\mathbb{E}^d$  whose Heesch number equals  $d - 1$ .

Part of this work is done jointly with A. Slivková.

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# Covering grids with multiplicity

Anurag Bishnoi  
TU Delft

12 Jul  
10:00am  
Section 1

Given finite sets  $S_1, \dots, S_d$  of a field  $\mathbb{F}$ , what is the smallest number of hyperplanes needed to cover every point of the grid  $\Gamma = S_1 \times \dots \times S_d$  at least  $k$  times, except one point that is not contained in any of the hyperplanes?

Ball and Serra proved that we need at least  $k(|S_1| - 1) + \sum_{i=2}^d (|S_i| - 1)$  hyperplanes, where  $S_1$  is the largest set. For  $k = 1$ , this bound recovers the Alon-Füredi theorem, which is in fact a special case of the classical Cayley-Bacharach theorem in algebraic geometry. If  $\mathbb{F} = \mathbb{F}_q$  and  $S_1 = \dots = S_d = \mathbb{F}_q$ , then this bound also implies the result of Bruen from 1992 on multiple blocking sets.

In this talk I will present new results on the tightness of the bound. In particular, we will see that if  $|S_1| \geq |S_2| \geq \dots \geq |S_d|$  and  $|S_1| \geq (k - 1)(|S_2| - 1) + 1$ , then the bound is always tight. Moreover, for  $d = 2$ , this appears to be the exact threshold at which the bound holds, in the sense that for almost all grids with  $|S_1| \leq (k - 1)(|S_2| - 1)$ , we prove a strictly larger lower bound. In higher dimensions, we prove tight asymptotic results for generic grids in every fixed dimension.

(Joint work with Simona Boyadzhyska, Aditya Potukuchi, and Shriyaa Srivastava)

# Two Recent Proofs of Theorems for matrices

Vladimir Blinovsky  
UNIFESP

8 Jul  
12:15pm  
Section 4

We introduce proofs of two theorems for matrices.

**Theorem 0.0.3** (*Brouwer's Conjecture*) Let  $G = (V, E)$  be graph with vertices  $V$ ,  $|V| = n$  and edges  $E$ ,  $|E| = m$ . Denote  $L(G) = A - D$  the Laplacian of graph  $G$ , where  $D = \text{diag}(d_i)$ ,  $A = a_{i,j}$  with  $a_{i,j} = -1$  if  $(i, j) \in E$  and  $a_{i,j} = 0$  otherwise, while  $d_i$  is degree of vertex  $i \in [n]$ . Let  $\mu_1 \geq \mu_2 \dots \mu_{n-1} \geq \mu_n = 0$  be eigenvalues of  $L(G)$ . Then the following inequality is valid

$$\sum_{i=1}^k \mu_i \leq m + \binom{n}{k}.$$

We prove this theorem for  $n > n_0 = 5 \cot 10^7$ , assuming that it is valid for  $n < n_0$ . Value  $n_0$  can be reduced by more precise calculations. Proof of this theorem is joint work with Llohan Speranca.

Second theorem which proof would like to introduce is the following.

**Theorem 0.0.4** Let  $A_n = \{a_{i,j}, i, j \in [n]\}$  be ensemble of Bernulli matrices,  $P(a_{i,j} = 1) = P(a_{i,j} = -1) = 1/2$ . Then

$$P(\det(A_n) = 0) = 4 \binom{n}{2} 2^{-n} (1 + o(1))$$

Using the same arguments we demonstrate validity of more precise asymptotic

$$P(\det(A_n) = 0) - 4 \binom{n}{2} 2^{-n} = 16 \binom{n}{4} \left(\frac{3}{8}\right)^n (1 + o(1)).$$

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## Clustering the Vertices of Sparse Edge-Weighted Graphs via Non-Backtracking Spectra

8 Jul  
11:50am  
Section 4

Marianna Bolla

Technical University Budapest

Füredi and Komlós (Combinatorica, 1981) characterized the distribution of the largest eigenvalue of a Wigner-type matrix, where the expectation of the independent entries was shifted with the same real number. More generally, their result was applied to the  $k$  structural adjacency eigenvalues of a  $k$ -block matrix, perturbed by a Wigner-type one (Bolla, Wiley, 2013). The so obtained random graphs are dense. If the random graph comes from a percolated stochastic  $k$ -block model (the probability that two vertices are connected independently depends on their cluster memberships and is decreasing with the number of vertices), a sparse graph sequence is obtained (with different scalings). In this case, seemingly contradicting to the laws of large numbers, the structural real eigenvalues of the non-backtracking matrix (and not of the adjacency matrix) are aligned with those of the expected adjacency matrix that has a good low-rank approximation. For this, the theory of deformed Wigner matrices (Capitaine, Donati–Martin, Féral, Ann. Prob., 2009) and recent results (Stephan, Massoulié, Math. Stat. Learning, 2022) are used. Furthermore, perturbation of the spectral subspaces corresponding to the structural eigenvalues is also investigated by means of Bauer–Fike type perturbation results instead of Davis–Kahan type ones. Since the squared distance between this spectral subspace and that of the  $k$ -block model graph is the objective function of the  $k$ -means algorithm, the representatives, based on the structural eigenvectors of the non-backtracking matrix, are able to recover the underlying clusters. With inflation–deflation techniques, we show how the corresponding eigenvectors of lower order companion matrices can be used to find assortative clusters of the vertices. Applications to quantum chemistry and social networks are also considered.

Contribution of Hannu Reittu (VTT Technical Research Center of Finland) and of former BSM student Daniel Zhou in making simulations and applications is acknowledged.

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## The growth rate of multicolor Ramsey numbers of 3-graphs

9 Jul  
10:25am  
Section 4

Domagoj Bradač

ETH Zürich

The  $q$ -color Ramsey number of a  $k$ -uniform hypergraph  $G$ , denoted  $r(G; q)$ , is the minimum integer  $N$  such that any coloring of the edges of the complete  $k$ -uniform hypergraph on  $N$  vertices contains a monochromatic copy of  $G$ . The study of these numbers is one of the most central topics in combinatorics. One natural question, which for triangles goes back to the work of Schur in 1916, is to determine the behaviour of  $r(G; q)$  for fixed  $G$  and  $q$  tending to infinity. In this paper we study this problem for 3-uniform hypergraphs and determine the tower height of  $r(G; q)$  as a function of  $q$ . More precisely, given a hypergraph  $G$ , we determine when  $r(G; q)$  behaves polynomially, exponentially or double-exponentially in  $q$ . This answers a question of Axenovich, Gyárfás, Liu and Mubayi.

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## A lower bound on the saturation number and a strengthening for triangle-free graphs

10 Jul  
11:15am  
Section 4

Calum Buchanan

University of Vermont

The saturation number  $sat(n, H)$  of a graph  $H$  and positive integer  $n$  is the minimum size of a graph of order  $n$  which does not contain a subgraph isomorphic to  $H$  but to which the addition of any edge creates such a subgraph. Erdős, Hajnal, and Moon first studied saturation numbers of complete graphs. In 2022, Cameron and Puleo determined a general lower bound on  $sat(n, H)$ . In this talk, we present another lower bound on  $sat(n, H)$ , with a strengthening when  $H$  is triangle-free. Demonstrating its effectiveness, we determine the saturation numbers of diameter-3 trees up to an additive constant; these are double stars  $S_{s,t}$  on  $s + t$  vertices whose centers have degrees  $s$  and  $t$ . Faudree, Faudree, Gould, and Jacobson proved that  $sat(n, S_{t,t}) = (t-1)n/2 + O(1)$ . We prove that  $sat(n, S_{s,t}) = (st+s)n/(2t+4) + O(1)$  when  $s < t$ . This talk is based on joint work with Puck Rombach.

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9 Jul  
10:25am  
Section 2

## A Hilton-Milner theorem for exterior algebras

Denys Bulavka

Hebrew University of Jerusalem

A set family  $F$  is pairwise-intersecting if every pair of its members intersect. In 1960, Erdős, Ko, and Rado gave an upper-bound on the size of a pairwise-intersecting family of  $k$ -sets coming from a ground set of size  $n$ . Moreover, they characterized the families achieving the upper-bound. These are families whose members all share exactly one element, so called trivial families. Later, Hilton and Milner provided the next best upper-bound for pairwise-intersecting families that are not trivial.

There are several generalizations of the above results. We will focus on the case when the set family is replaced with a subspace of the exterior algebra. In this scenario intersection is replaced with the wedge product, being pairwise-intersecting with self-annihilating and being trivial with being annihilated by a 1-form. Scott and Wilmer, and Woodrooffe gave an upper-bound on the dimension of self-annihilating subspaces of the exterior algebra. In the current work we show that the better upper-bound coming from Hilton and Milner's theorem holds for non-trivial self-annihilating subspaces.

This is a joint work with Francesca Gandini and Russ Woodrooffe.

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8 Jul  
11:25am  
Section 3

## Thresholds for Zero-Sum Sequences

Neal Bushaw

Virginia Commonwealth University

For an additive group  $\Gamma$  the sequence  $g_1, \dots, g_k$  of elements of  $\Gamma$  is a zero-sum sequence if  $g_1 + \dots + g_k = 0_\Gamma$ . Erdős-Ginzburg-Ziv initiated a new area of study by proving that for any positive integer  $n$ , every sequence of  $2n - 1$  elements from  $\mathbb{Z}/n\mathbb{Z}$  contains a zero-sum subsequence of length exactly  $n$ . The study of zero-sum subsequences has a long and rich history, with a particular focus on determining the length of sequence condition necessary to guarantee a length  $n$  zero-sum subsequence inside groups with specified properties. We explore this history, focusing on links to graph pebbling discovered by Chung and Lemke-Kleitman. We then explore a new randomized variation of this zero-sum setting, where the initial sequence is selected at random from a given group, and discuss thresholds in this setting. How long must we make our random sequence, to be 'reasonably' sure it contains a zero-sum subsequence of length  $n$ ? This makes use of recent threshold pebbling due to B. and Kettle; we also give a brief overview of this emergent area.

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# Reconfiguration of Independent Transversals

Pjotr Buys

University of Amsterdam

11 Jul  
11:15am  
Section 4

Given integers  $\Delta \geq 2$  and  $k \geq 2\Delta$ , suppose there is a graph of maximum degree  $\Delta$  and a partition of its vertices into parts of size at least  $k$ . By a seminal result of Haxell, there must be some independent set of the graph that is transversal to the parts, a so-called independent transversal. We show that, if moreover  $k \geq 2\Delta + 1$ , then every independent transversal can be transformed within the space of independent transversals to any other through a sequence of one-vertex modifications, showing connectivity of the so-called reconfiguration graph of independent transversals.

This is sharp in the sense that for  $k = 2\Delta$  (and  $\Delta \geq 2$ ) the connectivity conclusion fails, for instance for the graph  $K_{\Delta, \Delta}$  with a unique part. However, we also prove a stronger result for  $k = 2\Delta$ , a nearly complete characterization of those vertex-partitioned graphs for which the reconfiguration graph of independent transversals is connected.

It is based on joint work with Ross Kang and Kenta Ozeki:

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## A CHARACTERIZATION OF THE SEMITWIN DIGRAPH

Ofelia Cepeda-Camargo

Universidad Nacional Autónoma de México

12 Jul  
10:00am  
Section 3

Let  $D$  be a digraph,  $V(D)$  and  $A(D)$  denote the sets of vertices and arcs of  $D$ , respectively. We say that two different vertices  $u$  and  $v$  are semitwin vertices whenever  $N^-(u) = N^-(v)$  or  $N^+(u) = N^+(v)$ , where  $N^-(v) = \{x \in V(D) : (x, v) \in A(D)\}$  and  $N^+(v) = \{x \in V(D) : (v, x) \in A(D)\}$ . A digraph  $D$  is a semitwin digraph if and only if each pair of adjacent vertices in  $D$  are semitwins. In this talk we prove that if  $D$  is a semitwin strong digraph then  $D$  is a semicomplete digraph. A connected, not strong digraph is a semitwin digraph if and only if each strong component is a complete digraph and, its condensation digraph, is isomorphic to some induced subdigraph of a specific family.

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## Positroids are 3-colorable

Lamar Chidiac

Fernuniversitat in Hagen

Hadwiger's conjecture states that any graph which is not  $k$ -colorable must have a  $K_{k+1}$  minor. Using the definition of the chromatic number of oriented matroid introduced by J. Neseřil, R. Nickel, and W. Hochstattler, Hochstattler presented a generalisation of Hadwiger's conjecture to regular oriented matroids and proved that it is equivalent to Tutte's 4-flow and 5-flow conjectures when  $k = 4$  and  $k = 5$  respectively, and that it is equivalent to Hadwiger's conjecture for  $k \geq 6$ . The case  $k = 3$  remains the only non-trivial case. For general oriented matroid even the case  $k = 3$  remains open.

Goddyn, Hochstattler and Neudauer introduced the class of generalised series parallel (GSP) oriented matroids, which is an  $M(K_4)$ -free class and proved it is 3-colorable. If all  $M(K_4)$ -free oriented matroids are GSP, then Hadwiger's conjecture would hold for oriented matroids in the case of  $k = 3$ . We prove that positroids are GSP and therefore 3-colorable.

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## On $A$ -cordial trees

Sylwia Cichacz

AGH University, Cracow, Poland

Hovey introduced  $A$ -cordial labelings as a generalization of cordial and harmonious labelings. If  $A$  is an Abelian group, then a labeling  $f : V(G) \rightarrow A$  of the vertices of some graph  $G$  induces an edge labeling on  $G$ ; the edge  $uv$  receives the label  $f(u) + f(v)$ . A graph  $G$  is  $A$ -cordial if there is a vertex-labeling such that (1) the vertex label classes differ in size by at most one and (2) the induced edge label classes differ in size by at most one. In the literature, mostly cordial labeling in cyclic groups is studied. There is a famous (still open) conjecture which states that all trees are  $\mathbb{Z}_k$ -cordial for all  $k$ . The situation changes a lot if  $A$  is not cyclic. It was proved that all trees, except  $P_4$  and  $P_5$ , are  $\mathbb{Z}_2^2$ -cordial. Patrias and Pechenik posed a conjecture that for every group  $A$  there is an  $A$ -cordial labeling for almost every path. Erickson et al. extended the conjecture for all trees. In the talk, we show that the conjecture holds for paths, but it is not true for general trees - even if we consider an  $A$ -rainbow coloring instead of  $A$ -cordial (i.e. an  $A$ -cordial labeling in which  $|A| = |V(G)|$ ).

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# Recent Progress on the Odd Cover Problem

Alexander Clifton

Institute for Basic Science

11 Jul  
11:40am  
Section 2

In 1988, László Babai and Péter Frankl posed the “odd cover problem”, asking for the minimum number of complete bipartite graphs for which each edge of  $K_n$  is contained in an odd number of them. A natural lower bound is  $\frac{r_2(K_n)}{2}$  where  $r_2$  denotes the rank of the adjacency matrix over  $\mathbb{F}_2$ , and we call a collection of  $\frac{r_2(K_n)}{2}$  complete bipartite graphs with the desired property a *perfect odd cover* of  $K_n$ . We partially resolve the odd cover problem by showing the minimum is  $\frac{r_2(K_n)}{2} + 1$  for odd  $n$  and by demonstrating a new infinite family of even cliques which permit perfect odd covers. We also establish several properties which must be satisfied by any perfect odd cover of an even clique.

Joint work with Calum Buchanan, Eric Culver, Péter Frankl, Jiaxi Nie, Kenta Ozeki, Puck Rombach, and Mei Yin.

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# Chords in Longest Cycles

Luke Collins

University College London

11 Jul  
3:15pm  
Section 4

Let  $G$  be a graph on  $n$  vertices, and let  $C$  be a cycle in  $G$ . A *chord* in  $C$  is an edge in  $G$  between two vertices of  $C$  which does not already form part of the cycle. The following 1976 conjecture of Thomassen has garnered a lot of attention over the years:

**Conjecture 0.0.5** *Let  $G$  be 3-connected. Every cycle of maximum order in  $G$  has a chord.*

Thomassen himself proved that the conjecture holds in the case that  $G$  is cubic, and several other authors have looked at variations of the problem in various other classes of graphs; and in general these graphs are usually quite sparse. Indeed, for very dense graphs—even without any connectivity assumptions—the result is obvious, e.g., if  $\delta(G) \geq \frac{n}{2}$  then by Dirac’s theorem  $G$  is Hamiltonian and thus (provided  $n \geq 5$ ), any cycle of maximum order has a chord. On the other hand, the following construction

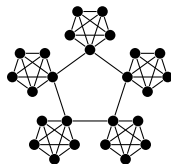


Figure 1: Illustration of Harvey’s construction with  $t = 5$ .

of Harvey shows that graphs having  $\delta(G) < \sqrt{n}$  can avoid chords: take  $t$  copies of  $K_t$ , and join them around an  $t$ -cycle, as illustrated in figure 1. This graph has  $\delta(G) = \sqrt{n} - 1$ , yet the central cycle is both of maximum length and chordless. Harvey conjectures that this bound is tight:

**Conjecture 0.0.6** *Let  $G$  be a graph on  $n$  vertices with  $\delta(G) > \sqrt{n} - 1$ . Every cycle of maximum order in  $G$  has a chord.*

In the same paper, he shows that this is true with the slightly stronger assumption that  $\delta(G) \geq \frac{3+\sqrt{17}}{2\sqrt{2}}\sqrt{n} \approx 2.52\sqrt{n}$ . We prove the following asymptotic form of the conjecture:

**Theorem 0.0.7** *Fix  $\epsilon > 0$ , and let  $G$  be an graph on  $n$  vertices satisfying  $\delta(G) \geq (1 + \epsilon)\sqrt{n}$ . Then if  $n$  is large enough, every cycle of maximum order in  $G$  has a chord.*

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9 Jul  
12:05pm  
Section 1

## Additive structure in convex translates

Gabriel Currier

University of British Columbia

Suppose we have a set  $P$  of  $n$  points in the plane, and another set  $S$  of  $n^{1/3}$  points in convex position. What can we say about the structure of this arrangement if  $P$  contains many translates of the set  $S$ ? I will discuss a recent result showing that if  $P$  contains around  $n$  translates of  $S$ , then the translation vectors must come from a generalized arithmetic progression of low dimension.

The motivation for this problem comes from incidence geometry, where many constructions for strictly convex curves having many incidences with pointsets follow this general outline. In particular, I will discuss applications to the Erdős unit distance conjecture. This is joint work with József Solymosi and Ethan White.

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8 Jul  
3:15pm  
Section 3

## Hamilton decompositions of regular tripartite tournaments

Francesco Di Braccio

London School of Economics

A regular  $k$ -partite tournament is a regular orientation of the balanced complete  $k$ -partite graph. Kühn and Osthus proved that for all  $k \geq 4$  any large enough regular  $k$ -partite tournament can be decomposed into edge-disjoint Hamilton cycles and conjectured the same to be true for all  $k \geq 2$ . Granet proved the conjecture for  $k = 2$  and, somewhat surprisingly, gave a construction of a non-Hamilton decomposable regular tripartite tournament. Our main result is a proof that an approximate version of the conjecture still holds for  $k=3$ , i.e. that in each such large tournament one may find edge-disjoint Hamilton cycles covering all but  $o(n^2)$  edges. We also prove that if  $a > 1/3$ , any balanced regular tripartite digraph with minimum semidegree at least  $a * n$  is Hamilton decomposable.

Based on joint work by Francesco Di Braccio, Joanna Lada, Viresh Patel, Yani Pehova, Jozef Skokan.

## Optimal Hamilton covers and linear arboricity for random graphs

Nemanja Draganic  
University of Oxford

10 Jul  
12:05pm  
Section 2

In his seminal 1976 paper, Pósa showed that for all  $p \geq C \log n/n$ , the binomial random graph  $G(n, p)$  is with high probability Hamiltonian. This leads to the following natural questions, which have been extensively studied: How well is it typically possible to cover all edges of  $G(n, p)$  with Hamilton cycles? How many cycles are necessary? In our paper we show that for  $p \geq C \log n/n$ , we can cover  $G \sim G(n, p)$  with precisely  $\lceil \Delta(G)/2 \rceil$  Hamilton cycles. Our result is clearly best possible both in terms of the number of required cycles, and the asymptotics of the edge probability  $p$ , since it starts working at the weak threshold needed for Hamiltonicity. This resolves a problem of Glebov, Krivelevich and Szabó, and improves upon previous work of Hefetz, Kühn, Lapinskas and Osthus, and of Ferber, Kronenberg and Long, essentially closing a long line of research on Hamiltonian packing and covering problems in random graphs. Joint work with Stefan Glock, David Munhá Correia and Benny Sudakov.

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## Some new results concerning polymorphic and $\sigma$ -morphic monotiles

Aleksa Džuklevski  
University of Novi Sad

11 Jul  
10:30am  
Section 4

We say that a tile (a simply connected, closed and bounded set in  $\mathbb{E}^2$ ) is  $m$ -morphic if it tiles the plane in  $m$  non-congruent ways. It is an open problem (asked by Grünbaum and Shephard in 1977) whether for each positive integer  $m$  there exists an  $m$ -morphic monotile (the record holder being an 11-morphic monotile discovered by Myers). It is also unknown if there exists a  $\sigma$ -morphic monotile (again asked by Grünbaum and Shephard, in 1981), which is a tile that tiles the plane in countable infinity of non-cogruent ways. Both of these problems can be asked in different settings, in which we either: i) allow tiles to be higher-dimensional; ii) permit tiles to be disconnected or iii) impose nongeometric matching rules on tiles (say via colored edges and rules on which colors can be placed against each other).

None of these problems were solved until recently. In this talk we will give a historical overview of the known results together with presenting solutions to all the variations except for the existence of a  $\sigma$ -morphic disconnected tile, which is still open. All the other questions are answered in the affirmative.

This is a joint work with Bojan Bašić and Anna Slivková.

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## Results on Bollobás set-pair systems

Yue Erfei  
Eötvös University

Suppose  $\mathcal{P} = \{(A_i, B_i) | i \in [m]\}$  is a family of pairs of sets, where  $A_i, B_i \subseteq [n]$ , and  $A_i \cap B_i = \emptyset$ . Then  $\mathcal{P}$  is called a Bollobás system if  $A_i \cap B_j \neq \emptyset$  when  $i \neq j$ , and a skew Bollobás system if  $A_i \cap B_j \neq \emptyset$  when  $i < j$ .

In 1965, to solve a problem on hypergraphs, Bollobás proved that for a Bollobás system  $\mathcal{P} = \{(A_i, B_i) | i \in [m]\}$ , we have  $\sum_{i=1}^m \binom{|A_i|+|B_i|}{|A_i|}^{-1} \leq 1$ , which is latter called Bollobás Theorem or Bollobás Inequality, and plays a important role in extremal set theory. If we further request  $|A_i| = a, |B_i| = b$  for every  $i$ , the inequality above shows that the maximum cardinality of the Bollobás system satisfies  $m \leq \binom{a+b}{a}$ . In 1977, Lovász generalized this (uniform) result to Bollobás system of spaces, and decided its maximum cardinality. In 1982, Frankl proved that both (uniform) statements remain true if the Bollobás system is replaced by skew Bollobás system. In this talk, we generalize the Bollobás-type theorem to partitions of sets. Using a polynomial method, we decide the maximum cardinality of skew Bollobás system on it. Also, using exterior product method, we consider the corresponding question of spaces, which is known as singular linear spaces. For the nonuniform case, we cannot simply weaken the condition of Bollobás system to skew Bollobás system as for the uniform case. So it is natural to ask what can we say for a (nonuniform) skew Bollobás system. In 2023, Hegedűs and Frankl answered the question with the inequality  $\sum_{i=1}^m \binom{|A_i|+|B_i|}{|A_i|}^{-1} \leq 1 + n$ . Using a probability method, we can strengthen their inequality to  $\sum_{i=1}^m (1 + |A_i| + |B_i|) \binom{|A_i|+|B_i|}{|A_i|}^{-1} \leq 1$ . And using similar arguments, we generalize this result to partitions of sets on both uniform and nonuniform cases.

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## Monochromatic infinite sets in Minkowski spaces

Nóra Frankl  
Open University

By a result of Erdős, Graham, Montgomery, Rothschild, Spencer, and Straus it is possible to colour the points of the  $d$ -dimensional Euclidean space with two colours such that there is no monochromatic isometric copy of a given infinite set. We prove several related results in other normed spaces. We show that the same is true in 2-dimensional  $\ell_p$  spaces. We also prove that if the unit ball of a normed plane is a polygon, then two colours are not enough. Further, we also show an example of an infinite set for which we need to colour with an arbitrarily large number of colours to avoid its monochromatic isometric copies in a space with the max norm of sufficiently large dimension. Joint work with Panna Gehér, Andrey Kupavskii, Arsenii Sagdeev and Géza Tóth.

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## Perfect colourings of regular graphs

Dirk Frettloeh  
Univ. Bielefeld

9 Jul  
10:25am  
Section 3

A vertex colouring of some graph  $G$  is called perfect if each vertex of colour  $i$  has exactly  $a_{ij}$  neighbours of colour  $j$ . E.g., if one red vertex is adjacent to one black vertex, one red vertex and two green vertices, then each red vertex in  $G$  is adjacent to one black vertex, one red vertex and two green vertices. Being perfect imposes several restrictions on the colour incidence matrix  $(a_{ij})$ . This talk surveys several (old and new) necessary conditions for a matrix to be the colour incidence matrix of a perfect colouring. As an application we determine all perfect colourings of the edge graphs of several regular polytopes in dimensions 3, 4 and 5 with two, three and four colours, respectively.

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## Defective Ramsey Numbers for Triangle-free Graphs

John Gimbel  
University of Alaska

9 Jul  
10:00am  
Section 4

A set of vertices is  $k$ -sparse if it induces a graph with maximum degree at most  $k$ . A set is  $k$ -dense if the set is  $k$ -sparse in the complement. We consider Ramsey numbers of the following type. For  $i, j$ , and  $k$ , we consider the smallest integer  $N$  where every graph of order  $N$  contains an  $i$ -set that is  $k$ -dense or a  $j$ -set that is  $k$ -sparse. In certain cases we limit this to the family of triangle-free graphs.

Joint work with Tinaz Ekim (Bogazici University, Istanbul) and Burak Erdem (Bogazici University, Istanbul).

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## Inducibility in the Hypercube

John Goldwasser  
West Virginia University

10 Jul  
11:15am  
Section 3

Let  $Q_d$  be the hypercube of dimension  $d$  and let  $H$  and  $K$  be subsets of the vertex set  $V(Q_d)$ , called configurations in  $Q_d$ . We say that  $K$  is an exact copy of  $H$  if there is an automorphism of  $Q_d$  which sends  $H$  onto  $K$ . Let  $H$  be a configuration in  $Q_d$  and let  $n \geq d$  be an integer. We let  $\lambda(H, d, n)$  be the maximum, over all configurations  $S$  in  $Q_n$ , of the fraction of sub- $d$ -cubes  $R$  of  $Q_n$  in which  $S \cap R$  is an exact copy of  $H$ , and we define the  $d$ -cube density  $\lambda(H, d)$  of  $H$  to be the limit as  $n$  goes to infinity of  $\lambda(H, d, n)$ . We have determined  $\lambda(H, d)$  exactly for six of the 14 configurations in  $Q_3$  (and have lower bound constructions very close to flag algebra computed upper bounds for six others) and several of the 238 configurations in  $Q_4$ , as well as for an infinite family of configurations. A couple of the lower bound constructions are quite miraculous. There are strong connections with the inducibility of graphs and lots of open questions.

Joint work with R. Hansen.

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8 Jul  
11:50am  
Section 1

## Some Upper Bounds for Property B

Karl Grill  
TU Wien

An  $n$ -uniform hypergraph is said to have property B if it admits a proper 2-coloring. We use various heuristic algorithms to obtain upper bounds on the size of the smallest hypergraph without property B for the case that  $n$  and the number of vertices are small, and we review constructive and probabilistic approaches to this question.

Joint with Daniel Linzmayer.

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11 Jul  
10:30am  
Section 1

## On arithmetic graphs

Lajos Hajdu  
University of Debrecen

Let  $S$  be a finite non-empty set of primes,  $\mathbb{Z}_S$  the ring of those rationals whose denominators are not divisible by primes outside  $S$ , and  $\mathbb{Z}_S^*$  the multiplicative group of invertible elements ( $S$ -units) in  $\mathbb{Z}_S$ . For a non-empty subset  $A$  of  $\mathbb{Z}_S$ , denote by  $G_S(A)$  the so-called arithmetic graph with vertex set  $A$  and with an edge between  $a$  and  $b$  if and only if  $a - b \in \mathbb{Z}_S$ . Arithmetic graphs and their Diophantine applications have been studied by many mathematicians, including Győry, Evertse, Stewart, Tijdeman, Leutbecher, Niklash, Ruzsa and others.

In the talk we discuss various representability problems of finite simple graphs  $G$  as  $G_S(A)$ . We show among others that for any finite graph  $G$  there exist infinitely many finite sets  $S$  of primes such that  $G$  can be represented with  $S$ . We deal with the question of infinite representability of graphs, i.e. when  $G$  can be represented in infinitely many, essentially different ways. In particular, we show that  $G$  is representable with every  $S$  if and only if  $G$  is cubical, i.e. embeddable in  $\{0, 1\}^n$  for some  $n$ . Besides combinatorial arguments, some deep Diophantine results concerning  $S$ -unit equations are also used in our proofs.

The presented results are joint with K. Győry and R. Tijdeman.

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9 Jul  
11:40am  
Section 2

## Generalized planar Turán numbers related to short cycles

Hilal Hama Karim  
Budapest University of Technology and Economics

Given graphs  $H$  and  $F$ , the generalised planar Turán number,  $ex_P(n, H, F)$ , is the maximum number of copies of  $H$  that an  $n$ -vertex  $F$ -free planar graph can contain. We investigate this function when  $H$  and  $F$  are short cycles. Namely, we find the exact value of  $ex_P(n, C_l, C_3)$ , where  $C_l$  is a cycle of length  $l$ , for  $4 \leq l \leq 6$ , and determine the extremal graphs in each case. Also, considering the reverse of these problems, we determine the value of  $ex_P(n, C_3, C_l)$ , for  $4 \leq l \leq 6$ .

Joint work with Ervin Győri.

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# Some problems and results on large acyclic sets in digraphs

12 Jul  
11:15am  
Section 3

Ararat Harutyunyan

University of Paris Dauphine

Given a digraph  $D$ , we denote by  $\vec{\alpha}(D)$  the maximum size of a subset  $S$  of vertices such that  $S$  induces an acyclic digraph. In this paper, we study  $\vec{\alpha}(D)$  in terms of the maximum degree of  $D$ . Our main result is that if  $D$  is a random  $r$ -regular digon-free digraph of order  $n$ , then  $\vec{\alpha}(D) = \Theta(n \log r/r)$ . This extends results of Spencer and Subramanian on Erdős-Rényi random digraph model. Our lower and upper bounds are a multiplicative factor of 4 away from each other. We also consider the coloring version of the problem. The dichromatic number  $\vec{\chi}(D)$  of a digraph  $D$  is the minimum number of induced acyclic sets that one can partition  $V(D)$ . We prove some results on  $\vec{\chi}(D)$ , one of which is an analogue of Bondy's theorem: every digraph of circumference  $s$  satisfies  $\vec{\chi}(D) \leq \lceil \frac{s}{2} \rceil$ . Along the way, we derive some related results and propose some conjectures, including a strengthening of the tournament reformulation of the Erdős-Hajnal conjecture.

Joint work with Colin McDiarmid and Gil Puig i Surroca.

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## A generalization of properly colored paths and cycles in edge-colored graphs

12 Jul  
11:15am  
Section 2

Felipe Hernandez-Lorenzana

National Autonomous University of Mexico

Consider the following edge-coloring of a graph  $G$ . Let  $H$  be a graph possibly with loops, an  $H$ -coloring of a graph  $G$  is defined as a function  $c : E(G) \rightarrow V(H)$ . We will say that  $G$  is an  $H$ -colored graph whenever we are taking a fixed  $H$ -coloring of  $G$ . A walk (path)  $(v_1, \dots, v_n)$  in an  $H$ -colored graph  $G$  is an  $H$ -walk ( $H$ -path) if and only if  $(c(v_1v_2), c(v_2v_3), \dots, c(v_{n-1}v_n))$  is a walk in  $H$ , and a cycle  $(v_1, \dots, v_n, v_1)$  is an  $H$ -cycle whenever  $(c(v_1v_2), c(v_2v_3), \dots, c(v_{n-1}v_n), c(v_nv_1), c(v_1v_2))$  is a walk in  $H$ . Hence,  $H$  decides which color transitions are allowed in a walk in order to be an  $H$ -walk. Notice that if  $H$  is the complete graph without loops, then a walk  $W$  is an  $H$ -walk if and only if  $W$  is a properly colored walk.

Let  $G$  be an  $H$ -colored graph. In this talk, we show sufficient conditions implying the existence of  $H$ -paths and  $H$ -cycles with certain length in  $G$ .

11 Jul  
11:40am  
Section 4

## Characterising flip process rules with the same trajectories

Eng Keat Hng

Czech Academy of Sciences

Garbe, Hladký, Šileikis and Skerman recently introduced a broad class of discrete-time random graph processes called flip processes. A flip process starts with a given  $n$ -vertex graph, and each step of a flip process involves randomly selecting a  $k$ -tuple of distinct vertices and modifying the induced subgraph according to a predetermined rule. Garbe, Hladký, Šileikis and Skerman showed that the typical behaviour of flip processes is neatly captured by certain continuous-time deterministic graphon trajectories. It was observed that seemingly different flip process rules may sometimes have the exact same collection of trajectories.

We obtain a complete characterisation of equivalence classes of flip process rules with the same trajectories. As an application, we show that the symmetric deterministic rules and the rules of order 2 are precisely the rules which are unique in their equivalence classes. These include complementing rules, component completion rules, clique removal rules, and extremist rules.

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11 Jul  
10:30am  
Section 2

## Triangle-free triple systems

Ron Holzman

Technion-Israel Institute of Technology

There are four non-isomorphic configurations of triples that can form a triangle in a 3-uniform hypergraph. Forbidding different combinations of these four configurations, fifteen extremal problems can be defined, several of which already appeared in the literature in some different context. Here we systematically study all of these problems solving the new cases exactly or asymptotically. In many cases we also characterize the extremal constructions.

It is joint work with Péter Frankl, Zoltan Füredi, Ido Goorevitch and Gábor Simonyi.

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# The maximum sum of the size of all intersections in intersecting families

Sumin Huang

8 Jul  
4:15pm  
Section 3

As one of the most fundamental theorems in extremal set theory, Erdős-Ko-Rado Theorem determined the upper bound of size of an intersecting  $k$ -family. This theorem has various generalizations, such as the Hilton–Milner theorem, the Ray-Chaudhuri–Wilson theorem, the  $r$ -wise intersection theorem, and the Complete Intersection theorem. The above-mentioned theorems all consider the maximum size of an intersecting family with some additional conditions. In this talk, instead of focusing on the size of the intersecting family, we consider the sum of size of all intersections of an intersecting family  $\mathcal{F}$ . Let  $\omega(\mathcal{F}) = \sum_{\{A,B\} \subset \mathcal{F}} |A \cap B|$ . Although the values of  $\omega(\mathcal{F})$  and  $|\mathcal{F}|$  are not directly correlated, by using cyclic permutation, we can still prove that when  $\mathcal{F}$  is a star,  $\omega(\mathcal{F})$  reaches its maximum value. Further, we generalize this result for crossing-intersecting families.

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# Recent results on the Holroyd-Talbot Conjecture

Glenn Hurlbert

Virginia Commonwealth University, USA

8 Jul  
3:15pm  
Section 2

In 2005 Holroyd and Talbot generalized the Erdős-Ko-Rado realm to graphs by restricting the family of all  $r$ -subsets of  $n$  elements under consideration to the family  $\mathcal{I}^r(G)$  of independent sets of size  $r$  in a graph  $G$  on  $n$  vertices. Say that a subfamily of  $\mathcal{I}^r(G)$  is a *star* if the intersection of its sets (its *center*) is nonempty. Let  $\mathcal{F}$  be an intersecting subfamily of  $\mathcal{I}^r(G)$  and denote the minimum size of a maximal independent set in  $G$  by  $\mu(G)$ . They conjectured that if  $r \leq \mu(G)/2$  then the size of  $\mathcal{F}$  is at most the size of some star.

After a brief history of earlier results by Deza-Frankl, Bollobás-Leader, and others, I will present more recent theorems and open problems with various collaborators including Feghali, Frankl, Kamat, and Meagher. Among the results are injective proofs of the Erdős-Ko-Rado and Hilton-Milner theorems, certification of the Holroyd-Talbot conjecture for smaller  $r$  on sparse graphs, and partial results and conjectures on trees regarding where the center of a largest star can be.

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# On Boolean Degree 1 Functions (Cameron-Liebler Sets) in Finite Vector Spaces

9 Jul  
10:00am  
Section 2

Ferdinand Ihringer

Southern University of Science and Technology (SUSTech),  
Shenzhen, China.

It is easy to see that if  $f$  is a real,  $n$ -variate affine function which is Boolean on the  $n$ -dimensional hypercube (that is,  $f(x) \in \{0, 1\}$  for  $x \in \{0, 1\}^n$ ), then  $f(x) = 0$ ,  $f(x) = 1$ ,  $f(x) = x_i$  or  $f(x) = 1 - x_i$ . The same classification holds if we restrict  $\{0, 1\}^n$  to elements with Hamming weight  $k$  if  $n - k, k \geq 2$ .

Let  $V(n, q)$  denote the  $n$ -dimensional vector space over the field with  $q$  elements. Since work by Cameron and Liebler in 1982, it has been asked if a similar classification holds for  $k$ -spaces in  $V(n, q)$ . It is known due to the work by Drudge (1998) and subsequent work that for  $(n, k) = (4, 2)$  such a classification is impossible. In our talk we will show that for fixed  $q, k \geq 2$  and  $n$  sufficiently large, a Boolean degree 1 function on the  $k$ -spaces of  $V(n, q)$  corresponds to one of the following:

1. The empty set.
2. All  $k$ -spaces through a fixed 1-space  $P$ .
3. All  $k$ -spaces in a fixed hyperplane  $H$ .
4. The union of the previous two examples when  $P$  is not in  $H$ .
5. The complement of any of the previous cases.

The result relies on the Ramsey theory for projective and affine spaces.

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## Dirac's theorem for linear hypergraphs

11 Jul  
3:15pm  
Section 2

Seonghyuk Im

KAIST / IBS ECOPRO

Dirac's theorem states that any  $n$ -vertex graph  $G$  with even integer  $n$  satisfying  $\delta(G) \geq n/2$  contains a perfect matching. We generalize this to  $k$ -uniform linear hypergraphs by proving the following. Any  $n$ -vertex  $k$ -uniform linear hypergraph  $H$  with minimum degree at least  $\frac{n}{k} + \Omega(1)$  contains a matching that covers at least  $(1 - o(1))n$  vertices. This minimum degree condition is asymptotically tight and obtaining perfect matching is impossible with any degree condition. Furthermore, we show that if  $\delta(H) \geq (\frac{1}{k} + o(1))n$ , then  $H$  contains almost spanning linear cycles, almost spanning hypertrees with  $o(n)$  leaves, and "long subdivisions" of any  $o(\sqrt{n})$ -vertex graphs.

This is joint work with Hyunwoo Lee.

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## The Turán Density for Daisies and Hypercubes

Maria-Romina Ivan  
University of Cambridge

9 Jul  
3:15pm  
Section 2

The Turán density of an  $r$ -uniform hypergraph  $H$ , denoted by  $\pi(H)$ , is the limit of the maximum density of an  $n$ -vertex  $r$ -uniform hypergraph not containing a copy of  $H$ , as  $n$  tends to infinity.

An  $r$ -daisy is an  $r$ -uniform hypergraph consisting of the six  $r$ -sets formed by taking the union of an  $(r - 2)$ -set with each of the 2-sets of a disjoint 4-set.

Bollobás, Leader and Malvenuto, and also Bukh, conjectured that the Turán density of the  $r$ -daisy is zero. A folklore Turán-type conjecture for hypercubes states that for fixed  $d$  the smallest set of vertices of the  $n$ -dimensional hypercube  $Q_n$  that meets every copy of  $Q_d$  has density  $1/(d + 1)$  as  $n$  goes to infinity.

In this talk, we show that the Turán density for daisies is positive, and, by adapting our construction, we also disprove the hypercube conjecture mentioned above.

Joint work with David Ellis and Imre Leader.

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## Quantitative Steinitz theorem and polarity.

Grigory Ivanov  
PUC Rio

8 Jul  
3:15pm  
Section 1

The quantitative Steinitz theorem asserts that if the convex hull of a subset of  $\mathbb{R}^d$  contains the standard unit ball  $B$ , then it is possible to choose  $2d$  points from the original set containing the ball  $rB$  for some  $r = r(d)$ . Recently, Marton Naszodi and I have obtained a polynomial bound on  $r$ . In this talk, we will explain how to utilize a certain polarity trick to effectively remove several points from the original set without drastically affecting the bound on  $r$ .

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## The Helly Property for the Hamming Balls

Zhihan Jin  
ETH Zurich

11 Jul  
12:05pm  
Section 3

The celebrated Helly's theorem states that given any finite collection of convex subsets in  $\mathbb{R}^d$ , if every  $d + 1$  of them intersects, then the whole collection has a nonempty intersection. It has a lot of applications and extensions. We consider the similar question for Hamming balls of radius  $t$  in  $[X]^n$ , for any alphabet  $X$ . We show that if any  $2^{t+1}$  of such balls in the family intersects, then they all have a nonempty intersection. This is the best possible and answers a question of Raman, Subedi, and Tewari.

Joint work with N. Alon and B. Sudakov.

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10 Jul  
11:40am  
Section 4

## Rainbow Saturation

Daniel Johnston  
Trinity College

A graph  $G$  is rainbow  $H$ -saturated if there is some proper edge coloring of  $G$  which is rainbow  $H$ -free (that is, it has no copy of  $H$  whose edges are all colored distinctly), but where the addition of any edge makes such a rainbow  $H$ -free coloring impossible. Taking the maximum number of edges in a rainbow  $H$ -saturated graph recovers the rainbow Turán numbers whose systematic study was begun by Keevash, Mubayi, Sudakov, and Verstraete. In this talk, we introduce and examine the corresponding rainbow saturation number – the minimum number of edges among all rainbow  $H$ -free graphs.

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11 Jul  
12:05pm  
Section 3

## The Lovász-Cherkassky theorem in infinite graphs

Attila Joó  
University of Hamburg

Lovász and Cherkassky independently discovered in the 1970s that if  $G$  is a finite graph with a given set  $T$  of terminal vertices, such that  $G$  is inner Eulerian with respect to  $T$ , then the maximal number of edge-disjoint paths connecting distinct vertices in  $T$  is  $\frac{1}{2} \sum_{t \in T} \lambda(t, T - t)$ , where  $\lambda$  is the local edge-connectivity function. The optimality of a system of edge-disjoint  $T$ -paths in the Lovász-Cherkassky theorem is witnessed by the existence of certain cuts by Menger's theorem.

The infinite generalization of Menger's theorem by Aharoni and Berger (formerly known as the Erdős-Menger Conjecture), together with the characterization of infinite Eulerian graphs due to Nash-Williams, indicate what a structural infinite generalization of the Lovász-Cherkassky theorem should be. We present this generalization, some key ideas of the proof, and the arising difficulties compared to the finite case.

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8 Jul  
11:50am  
Section 2

## The Quantitative Fractional Helly theorem

Attila Jung  
Eötvös University

Two celebrated extensions of Helly's theorem are the *Fractional Helly theorem* of Katchalski and Liu (1979) and the *Quantitative Volume theorem* of Bárány, Katchalski, and Pach (1982). Improving on several recent works, we prove an optimal combination of these two results. We show that given a family  $\mathcal{F}$  of  $n$  convex sets in  $\mathbb{R}^d$  such that at least  $\alpha \binom{n}{d+1}$  of the  $(d+1)$ -tuples of  $\mathcal{F}$  have an intersection of volume at least 1, then one can select  $\Omega_{d,\alpha}(n)$  members of  $\mathcal{F}$  whose intersection has volume at least  $\Omega_d(1)$ . Joint work with Nóra Frankl and István Tomon.

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## Metric spaces with many degenerate triangles

Ida Kantor

Charles University, Prague

9 Jul  
11:15am  
Section 1

Let  $(X, d)$  be a metric space. A triple  $\{a, b, c\}$  of points of  $X$  is called a *degenerate triangle* if  $d(a, c) = d(a, b) + d(b, c)$ . Richmond and Richmond (1997) proved the following theorem: If, in a metric space with at least five points, all triangles are degenerate, then the space is isometric to a subset of the real line. We prove that the hypothesis is unnecessarily strong: In a metric space on  $n$  points, fewer than  $(7n^2)/6$  suitably placed degenerate triangles suffice. However, fewer than  $n(n-1)/2$  degenerate triangles, no matter how cleverly placed, never suffice.

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## Fourier analysis modulo $p$ on the Boolean cube.

Thomas Karam

University of Oxford

9 Jul  
11:15am  
Section 4

Fourier analysis in theoretical computer science is most commonly defined on the Boolean cube  $\{0, 1\}^n$  identified with  $\mathbb{Z}_2^n$ . Recently, generalisations of that setting have been studied, involving restrictions to  $\{0, 1\}^n$  of characters modulo  $p$  for some arbitrary prime  $p$ . Unlike in the case of mod-2 characters, different mod- $p$  characters are no longer orthogonal when restricted to the Boolean cube. We will begin by discussing some of the basic phenomena involved, and then briefly mention some recent results and remaining difficulties.

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## Hitting and coloring subsets in geometric hypergraphs

Chaya Keller

Ariel University

9 Jul  
11:15am  
Section 4

Problems on hitting sets (a.k.a. transversals) and on coloring are abundant in (hyper)graph theory and its applications. Besides the "mainstream" works on hitting and coloring vertices, various papers studied (either explicitly or implicitly) the setting where  $t$ -subsets of vertices are colored or hit. We will briefly present a few recent results in this direction, including a new approach to Zarankiewicz's problem via hitting  $t$ -subsets. More interestingly, we will concentrate on open problems motivated by these results.

Based on joint works with Bruno Jartoux, Shakhar Smorodinsky, and Yelena Yuditsky.

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## Violator and Co-Violator Spaces

Yulia Kempner

Holon Institute of Technology

A notion of a violator space was introduced by Matoušek et al. in 2008 as a combinatorial framework that encompasses linear programming and other geometric optimization problems. A violator space is a pair  $(E, \Phi)$ , where  $E$  is a finite set and  $\Phi$  is an operator  $2^E \rightarrow 2^E$  such that for all subsets  $X, Y \subseteq E$  the following properties are satisfied:

V1:  $X \subseteq \Phi(X)$  (extensivity),

V2:  $(X \subseteq Y \subseteq \Phi(X)) \Rightarrow \Phi(X) = \Phi(Y)$  (self-convexity).

We introduce co-violator spaces based on contracting operators known also as choice functions. A co-violator space is a pair  $(E, c)$ , where  $E$  is a finite set and  $c$  is an operator  $2^E \rightarrow 2^E$  such that for all subsets  $X, Y \subseteq E$  the following properties are satisfied:

CV1:  $c(X) \subseteq X$  (contracting property),

CV2:  $(c(X) \subseteq Y \subseteq X) \Rightarrow c(X) = c(Y)$  (outcast property).

For an arbitrary space  $(E, \alpha)$  with the operator  $\alpha : 2^E \rightarrow 2^E$ , an element  $x$  of a subset  $X \subseteq E$  is an extreme point of  $X$  if  $x \notin \alpha(X - x)$ . The set of extreme points of  $X$  is denoted by  $ex(X)$ . We demonstrate that if  $\alpha$  is a violator operator of a uniquely generated violator space, then  $ex$  is a co-violator operator. Given an operator, say  $\sigma$ , two sets  $X$  and  $Y$  are cospanning if  $\sigma(X) = \sigma(Y)$ . Cospanning characterizations of violator spaces allow us to obtain some new properties of violator operators and co-violator operators, emphasizing their interconnections. In particular, we show that uniquely generated violator spaces satisfy so-called Krein-Milman properties, i.e.,  $\Phi(ex(X)) = \Phi(X)$  and  $ex(\Phi(X)) = ex(X)$  for every  $X \subseteq E$ .

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## Solutions to the discrete Pompeiu problem and to the finite Steinhaus tiling problem

Gergely Kiss

HUN-REN Alfréd Rényi Institute of Mathematics

Let  $K$  be a nonempty finite subset of the Euclidean space  $\mathbb{R}^k$  ( $k \geq 2$ ). In this talk we discuss the solution of the following so-called discrete Pompeiu problem: Is it true that, if a function  $f : \mathbb{R}^k \rightarrow C$  is such that the sum of  $f$  on every congruent copy of  $K$  is zero, then  $f$  vanishes everywhere? In fact, we solve a stronger, weighted version of this problem. As a corollary we obtain that every finite subset of  $\mathbb{R}^k$  having at least two elements is a Jackson set; that is, no subset of  $\mathbb{R}^k$  intersects every congruent copy of  $K$  in exactly one point.

This is a joint work with Miklós Laczkovich.

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## Small weakly separating path systems for complete graphs

George Kontogeorgiou  
University of Chile

11 Jul  
4:05pm  
Section 3

Let  $G$  be a graph and let  $P$  be a set of paths of  $G$ . We say that  $P$  weakly separates  $G$  if for every pair of edges of  $G$  there exists a path in  $P$  that contains exactly one of them. It is a well-known problem to determine the size of the smallest weakly separating path system of a given graph on  $n$  vertices. Around a decade ago, Falgas-Ravry, Kittipassorn, Korandi, Letzter and Narayanan conjectured an upper bound of  $O(n)$  paths. This was proved last year by Bonamy, Botler, Dross, Naia and Skokan, who further conjectured an upper bound of  $(1 + o(1))n$  paths.

Some authors have considered the restriction of this problem to complete graphs. It is known that a weakly separating path system for a complete graph on  $n$  vertices must contain at least  $n - 1$  paths. Recently, Fernandes, Oliveira Mota and Sanhueza-Matamala proved that  $(1 + o(1))n$  paths suffice.

In recent work with Maya Stein, we proved that every complete graph on  $n$  vertices has a weakly separating path system of  $n + 2$  paths. I will provide a brief view of the history of the problem and a proof sketch.

---

## Universal graphs with forbidden minors

Thilo Krill  
University of Hamburg

9 Jul  
4:05pm  
Section 2

A universal graph in a class of graphs  $\mathcal{G}$  is a graph  $G \in \mathcal{G}$  such that every graph in  $\mathcal{G}$  is isomorphic to an induced subgraph of  $G$ . A famous open problem is to decide for which finite connected graphs  $X$  there is a universal graph in the class of all countable graphs excluding  $X$  as a subgraph. Notable contributions to this problem have been made by, among others, Zoltán Füredi and János Pach.

In this talk we look at a variation of this problem: Let  $X$  be any finite connected graph. We conjecture that there exists a universal graph in the class of all countable graphs excluding  $X$  as a minor if and only if  $X$  is planar. We confirm this conjecture in the case where  $X$  is a wheel.

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## Uniform Turán densities of 3-uniform hypergraphs.

Filip Kučerák  
Masaryk University

12 Jul  
11:15am  
Section 1

In the 1980s, Erdős and Sós introduced the notion of uniform Turán density, where in addition to the classical notion of Turán density, the edges of the host graph are required to be distributed uniformly. Until recently, there was no 3-graph with a non-zero uniform Turán density whose uniform Turán density would be determined exactly. We now know constructions of 3-graphs with uniform Turán density equal to  $1/27$ ,  $4/27$ , and  $1/4$ . During the talk, we will present constructions of hypergraphs with additional non-zero values of uniform Turán density.

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## Orientations of graphs with at most one directed path between every pair of vertices

12 Jul  
10:25am  
Section 3

Gaurav Kucherya  
Charles University

Given a graph  $G$ , we say that an orientation  $D$  of  $G$  is a KT orientation if, for all pairs of vertices,  $u, v$  of  $D$ , there is at most one directed path (in any direction) between  $u$  and  $v$ . Graphs that admit such orientations have been used by Kierstead and Trotter (1992), Carbonero, Hompe, Moore, and Spirkl (2023), Briański, Davies, and Walczak (2023), and Girão, Illingworth, Powierski, Savery, Scott, Tamitegami, and Tan (2024) to construct graphs with large chromatic number and small clique number that served as counterexamples to various conjectures.

Motivated by this, we consider which graphs admit KT orientations (named after Kierstead and Trotter). In particular, we construct a graph family with small independence number (sub-linear in the number of vertices) that admits a KT orientation. We show that the problem of determining whether a given graph admits a KT orientation is NP-complete, even if we restrict ourselves to planar graphs. Finally, we provide an algorithm to decide if a graph with maximum degree at most 3 admits a KT orientation. Whereas, for graphs with maximum degree 4, the problem remains NP-complete.

This is joint work with Barbora Dohnalová, Jiří Kalvoda and Sophie Spirkl.

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## Counterexamples to the thrackle conjecture on higher genus surfaces

8 Jul  
12:25pm  
Section 2

Jan Kyncl  
Charles University

The thrackle conjecture for nonplanar surfaces, formulated by Cairns and Nikolayevsky, states that if a graph  $G = (V, E)$  has a thrackle drawing on an orientable surface  $S_g$  of genus  $g > 0$ , then  $|E| - |V| \leq 2g$ . We disprove this conjecture, showing that for each  $g > 0$  there is a graph  $G = (V, E)$  with  $|E| - |V| = 3g$  that can be thrackled on  $S_g$ .

Joint work with César Hernández-Vélez, Gelasio Salazar and Kebin Velasquez

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## Degree powers in graphs forbidding broom forests and double brooms

Ting Lan

Sun Yat-sen University

9 Jul  
11:15am  
Section 2

For a graph  $G$  with degree sequence  $d_1, \dots, d_n$ , and for a positive integer  $p$ , let  $e_p(G) = \sum_{i=1}^n d_i^p$ . In 2000, Caro and Yuster [A Turán type problem concerning the powers of the degrees of a graph, *Electron. J. Combin.* 7 (2000), R47] introduced the following Turán type problem: Given a positive integer  $p$  and a graph  $H$ , determine the function  $ex_p(n, H)$ , which is the maximum value of  $e_p(G)$  taken over all graphs  $G$  on  $n$  vertices that do not contain  $H$  as a subgraph. Obviously, we have  $ex_1(n, H) = 2ex(n, H)$ , where  $ex(n, H)$  denotes the classical Turán function.

Previous results on this problem, obtained by various authors, include the determination of the function  $ex_p(n, H)$  when  $H$  is a complete graph, a cycle, a linear forest, a star forest, and a broom (a broom is a path with a star at one end). In this talk, we shall present some new results for the function  $ex_p(n, H)$  when  $H$  is a broom forest, and a double broom (a double broom is a path with stars at both ends, with a double star being a special case). It is interesting to note that, for the case when  $H$  is a double broom, whose central path has even length, and the two stars are equal in size, the problem of the determination of  $ex_p(n, H)$  is related to the well studied degree-diameter problem.

This talk is based on joint work with Henry Liu (Sun Yat-sen University) and Rongxia Tang (Sun Yat-sen University).

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## Honeycomb conjecture in normed planes

Zsolt Lángi

Budapest University of Technology and Economics

8 Jul  
4:40pm  
Section 1

The Honeycomb Conjecture states that among tilings with unit area cells in the Euclidean plane, the average perimeter of a cell is minimal for a regular hexagonal tiling. This conjecture goes back to a book of the Roman polyhistor Varro, but it was proved only in the middle of the 20th century by L. Fejes Toth for convex tilings, and in the beginning of the 21th century by Hales for not necessarily convex tilings. It seems a natural question to ask whether in any normed plane, among tilings with unit area cells, the average perimeter is minimal for a tiling whose cells are translates of a given (not necessarily regular) hexagon. In this talk we investigate this question for convex tilings in normed planes. We show that in this case the answer is affirmative in any normed plane, if we replace average perimeter by average squared perimeter, and show that the original problem in general is related to an alpha-convex variant of Dowker's theorem on the areas of polygons circumscribed about a plane convex body. We use this connection to prove results about both problems. Joint work with Shanshan Wang.

8 Jul  
3:15pm  
Section 4

## Conjectures for Paley Graphs

Craig Larson

Virginia Commonwealth University

Paley graphs are defined on  $4k+1$  primes  $p$ . The vertices are  $0, \dots, p-1$ , where  $vw$  is an edge if  $v$  and  $w$  differ by a quadratic residue in the field with  $p$  elements. In investigating the independence structure of these graphs, we define the subgraphs  $Q_+$  and  $Q'$ ; where  $Q_+$  is the subgraph induced on the non-zero vertices adjacent to 1, and where  $Q'$  is the subgraph induced on the vertices not adjacent to 1. We can prove counts of various quantities in these subgraphs (including triangles).

More interestingly, we conjecture formulas for various counts involving the summands from the theorem of Fermat and Euler that  $p$  can be represented as  $x^2 + y^2$ . For instance, we conjecture that the number of edges in  $Q_+$  is  $\frac{k^2-5k+4}{4} + \frac{y^2-1}{16}$ . These conjectures have been verified for all  $4k+1$  primes up to 2917.

This is joint work with Robert Jacobs.

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8 Jul  
4:15pm  
Section 2

## Combinatorial transverse intersection algebra

Ruth Lawrence-Naimark

Hebrew University, Jerusalem

According to folklore, it is impossible to construct a faithful finite dimensional algebraic model of differential forms which preserves all three properties of (graded) commutativity, associativity and the Leibniz rule. In this talk we will demonstrate how by enlarging a cubical complex by adding certain "ideal" elements, a combinatorial transverse intersection algebra model of a torus can be constructed which does have graded commutativity and associativity while the product rule holds for elements of the original complex. One application of this algebra is to create a finite dimensional fluid algebra which can be implemented numerically for approximation to Euler's equation on a torus.

This is joint work with Daniel An and Dennis Sullivan.

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## Disconnected common graphs via supersaturation

Jae-baek Lee  
University of Victoria

8 Jul  
4:40pm  
Section 4

A graph  $H$  is said to be *common* if the number of labelled monochromatic copies of  $H$  in a 2-colouring of the edges of a large complete graph is asymptotically minimized by a random colouring. It is well known that the disjoint union of two common graphs may not be common; e.g.,  $K_2$  and  $K_3$  are common, but their disjoint union is not. We find the first pair of uncommon graphs whose disjoint union is common and a common graph and an uncommon graph whose disjoint union is common. Our approach is to reduce the problem of showing that certain disconnected graphs are common to a constrained optimization problem, in which the constraints are derived from supersaturation bounds related to Razborov's Triangle Density Theorem. In addition, we also improve the upper bound of Ramsey multiplicity constant of a triangle with a pendant edge and the disjoint union of  $K_3$  and  $K_2$ . This is joint work with Jonathan Noel.

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## Coloring Hypergraphs

Nathan Lemons  
Los Alamos National Laboratory

11 Jul  
3:40pm  
Section 2

A well known problem from an excellent book of Lovász states that any hypergraph with the property that no pair of hyperedges intersect in exactly one vertex can be properly 2-colored. Motivated by this as well as recent works of Keszegh and of Gyárfás et al we study the intersection graph of a hypergraph. The intersection graph encodes those pairs of hyperedges in a hypergraph that intersect in exactly one vertex. We prove all hypergraphs whose intersection graph is bipartite can be properly 2-colored. Joint work with Zoltán Blázsik.

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## On the existence of $(r, g, \chi)$ -graphs and cages

Linda Lesniak  
Western Michigan University

9 Jul  
10:00am  
Section 3

In 1961, Erdős showed the following: Given integers  $\chi \geq 3$  and  $g \geq 3$ , there exists a graph with chromatic number  $\chi$  and girth  $g$ . In 1947, Tutte asked the following question: Given integers  $r \geq 2$  and  $g \geq 3$ , does there exist an  $r$ -regular graph with girth  $g$ ? Erdős and Sachs established existence for all pairs  $r, g$  in 1960. Finally, Rubio-Montiel considered the problem of the existence of graphs with a given regularity  $r$  and chromatic number  $\chi$ . In all of these cases, the required graphs of minimum order are referred to as cages.

In this talk we'll look at the question of the existence of  $(r, g, \chi)$ -graphs, that is,  $r$ -regular graphs of girth  $g$  with chromatic number  $\chi$ , and the smallest such graphs

will be called  $(r, g, \chi)$ -cages. These graphs were introduced by Gabriela Araujo-Pardo, Zhanar Berikkyzy and Linda Lesniak, where the emphasis was on the case  $\chi = 3$ . But we now know, for example, according to work of Gabriela Araujo-Pardo, Julio Diaz-Calderon, Julian Fresan, Diego Gonzales-Moreno, Linda Lesniak and Mika Olsen, that if  $g$  and  $\chi$  are integers both at least 3, then for  $r$  sufficiently large there exist  $(r, g, \chi)$ -graphs.

Finally, we'll consider the question of the existence of "equitable"  $(r, g, \chi)$ -graphs and cages, that is,  $(r, g, \chi)$ -graphs with a  $\chi$ -coloring in which the color classes differ in size by at most 1, and those of minimum order.

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9 Jul  
11:40am  
Section 3

## On the edge-color index of rainbow subgraphs

Binlong Li

Northwestern Polytechnical University, Xi'an

Let  $G$  and  $H$  be two graphs. The Turán number  $ex(G, H)$  is the maximum number of edges of a subgraph of  $G$  without copies of  $H$ ; and the anti-Ramsey number  $ar(G, H)$  is the maximum number of colors of an edge-coloring of  $G$  without rainbow copies of  $H$ . We define the edge-color index of  $H$  in  $G$ , denoted by  $ec(G, H)$ , as the maximum sum of numbers of edges and colors of an edge-coloring of a subgraph of  $G$  without rainbow copies of  $H$ , i.e.,

$$ec(G, H) = \max\{e(F) + c(F) : F \subset G \text{ and } F^c \text{ is rainbow } H\text{-free}\}.$$

One can easily get that  $ec(G, H) = \max\{e(F) + ar(F, H) : F \subset G\}$ . In this talk, we will study the edge-color index of triangle and quadrangle in complete graphs, complete bipartite graphs or complete tripartite graphs.

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9 Jul  
3:15pm  
Section 4

## Antimagic Labeling for Subdivisions of Graphs

Wei-Tian Li

National Chung Hsing University

An antimagic labeling of a graph  $G$  with  $m$  edges is a bijection between the edge set of  $G$  and  $\{1, 2, \dots, m\}$  such that when summing up the label of each edge incident to the same vertex, different vertices will have different sums. A graph  $G$  with such a labeling is said to be antimagic. It was conjectured by Hartesfield and Ringel that every connected graph other than an edge is antimagic.

In this talk, I will present my recent results on the antimagic problem for the subdivisions of graphs. By  $G^{(s)}$ , we mean the graph obtained by replacing each edge of  $G$  with a path on  $s$  edges. For various types of graphs, we give the conditions on the minimum degree of  $G$  and the number  $s$  to show that  $G^{(s)}$  is antimagic. Particularly, when  $G$  is a complete graph or a complete bipartite graph, we show that  $G^{(s)}$  is antimagic for all  $s \geq 2$ .

Some different versions of the antimagic problem of graphs and the corresponding results will be presented as well.

# Graphon Branching Processes and Fractional Isomorphism

Anna Margarethe Limbach  
Czech Academy of Sciences

11 Jul  
12:05pm  
Section 4

In their study of the giant component in inhomogeneous random graphs, Bollobás, Janson, and Riordan introduced a class of branching processes parametrized by an  $L^1$ -graphon. We prove that two such branching processes have the same distribution if and only if the corresponding graphons are fractionally isomorphic, a notion introduced by Grebík and Rocha.

A different class of branching processes was introduced Hladký, Nachmias, and Tran in relation to uniform spanning trees in finite graphs approximating a given graphon. We characterize which graphons yield the same distribution.

This talk is based on joint work with Eng Keat Hng and Jan Hladký.

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## Rainbow cycles through specified vertices

Henry Liu  
Sun Yat-sen University

11 Jul  
4:05pm  
Section 4

An edge-coloured graph is rainbow if its edges are given distinct colours. Let  $1 \leq k \leq n$ , and  $G$  be a graph of order  $n$  with the property that any  $k$  vertices lie on a cycle. For example,  $G$  has this property if it is Hamiltonian or  $k$ -connected. Let  $crx_k(G)$  denote the minimum number of colours in an edge-colouring of  $G$  so that, any  $k$  vertices lie on a rainbow cycle. The problem of the determination of the parameter  $crx_k(G)$  can be classed into at least the following three active research directions:

- Edge chromatic numbers, since  $crx_k(G)$  can be considered as a type of edge chromatic number.
- The study of rainbow coloured subgraphs of a graph.
- The study of cycles through specified vertices in a graph.

We will present several results regarding the parameter  $crx_k(G)$ . For  $k = 1, 2, n$ , we will characterise the graphs  $G$  such that  $crx_k(G) = e(G)$ , and consider this problem for the case  $k = n - 1$ , which has some interesting relations with hypohamiltonian graphs. We will also compute or estimate the function  $crx_k(G)$  for specific graphs  $G$ , including wheels, complete graphs, complete bipartite and multipartite graphs, and discrete cubes. Various methods are employed to prove the results, including ideas from the theories of  $k$ -connected and  $k$ -linked graphs, and rainbow matchings, as well as the use of standard probabilistic methods.

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12 Jul  
10:25am  
Section 4

## Supercongruences and MC-finiteness of Integer Sequences.

Janos A. Makowsky

Computer Science, Technion-IIT, Haifa, Israel

An integer sequence  $s(n)$  is C-finite if it satisfies a linear recurrence relation. It is MC-finite if for every modulus  $m$  this is the case. If this is true for almost all  $m$  some authors speak of supercongruences. There are essentially three ways of showing that  $s(n)$  is MC-finite: By direct inspection, by model theoretic means (pioneered by E. Specker and C. Blatter in 1981) and by recursion theoretic means suggested by E. Specker and made precise more recently. In this talk we discuss these methods and give non-trivial examples and applications. (Based on recent work by Y Filmus, E Fischer, JA Makowsky, V Rakita)

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9 Jul  
4:05pm  
Section 1

## Counting antichains in the Boolean lattice

Alexandru Malekshahian

King's College London

An old question of Dedekind asks for the number of antichains in the Boolean lattice on  $n$  elements. After a long series of increasingly precise results, Korshunov determined this number up to a multiplicative factor of  $(1 + o(1))$ . We revisit Dedekind's problem and study the typical structure of antichains using tools from probability and statistical physics. This yields a number of results which include refinements of Korshunov's asymptotics and a 'sparse' version of Sperner's Theorem.

Joint work with Matthew Jenssen and Jinyoung Park.

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10 Jul  
12:05pm  
Section 3

## " $(j \times \text{ith prime}) \text{ adj } (i \times \text{jth prime})$ ": The Fundamental Relation of Arithmetic

David Matula

SMU

For all  $i, j$  let the relation "adj" assign both an arc labeled  $p_{sub i}$  directed into a node labeled  $n = (j \times p_{sub i})$  and an arc labeled  $p_{sub j}$  directed into a node labeled  $m = (i \times p_{sub j})$ . We observe that the resulting network has the multiset of arcs directed into any node providing the unique factorization of that node's label. All integers occur as node labels, appearing unique to isomorphism. This result motivates the term "fundamental relation of arithmetic" following the term Fundamental Theorem of Arithmetic ascribed to Euclid's unique factorization theorem. The underlying graph of this network is noted to be the forest of all finite trees, providing a visual format for Euclid's result.

If we extend this adjacency relation to a connectivity relation we have an equivalence relation where the equivalence classes of natural numbers are in one-to-one correspondence with the family of all finite trees. A practical result is that this allows the forest of all finite trees to be naturally ordered by the minimum members of corresponding equivalence classes.

Joint work with A. MacCarthy.

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## Combinatorial Piercing the Chessboard

Viola Mészáros  
Berlin

10 Jul  
12:05pm  
Section 1

Ambrus, Bárány, Frankl and Varga considered the minimum number of lines to pierce all the cells of the  $n \times n$  chessboard. They show it is between  $0.7n$  and  $n - 1$ , for  $n > 2$ , where the upper bound is conjectured to be sharp. We consider this geometrical problem in a combinatorial setting.

A combinatorial line is a sequence of cells, where consecutive cells share a side on the chessboard and it is monotone by both of the axes. A line is steep if it does not contain more than 2 consecutive cells of the same row (horizontal) of the chessboard. A plateau is a line consisting of three parts: vertical start, horizontal middle and vertical end. The name of the plateau is inspired by its mid-section.

We show that  $n$  steep combinatorial lines are needed to pierce the chessboard. We also show that if we have steep lines and plateaus, then at least  $0.75n$  of them are needed to pierce the chessboard.

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## Point-variety incidences over arbitrary fields

Aleksa Milojević  
ETH Zurich

9 Jul  
10:25am  
Section 1

In this talk I will discuss the problem of estimating the number of incidences of points and  $d$ -dimensional varieties of bounded degree over arbitrary fields, under the standard non-degeneracy assumption that no  $s$  varieties have  $s$  points in common. For a set of  $m$  points and  $n$  varieties, we determine the maximum number of incidences up to a constant factor as a function of  $m$  and  $n$ . I will present two different approaches to this problem, one based on the novel framework of induced Turán-type problems and the other based on VC-dimension theory. In particular, we extend the celebrated result of Rónyai, Babai and Ganapathy on the number of zero-patterns of polynomials to the context of varieties.

This talk is based on joint work with Benny Sudakov and István Tomon.

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## Plane colorings and arithmetic progressions

Kenneth Moore  
University of British Columbia

11 Jul  
3:40pm  
Section 1

A conjecture of Erdős, Graham, Montgomery, Rothschild, Spencer and Straus states that, with the exception of equilateral triangles, any two-coloring of the plane will have a monochromatic congruent copy of every three-point configuration. In this presentation, we will discuss the recent proof of one of the most natural open cases, namely that any two-colouring of the plane admits a monochromatic congruent copy of every 3-term arithmetic progression.

This talk is based on a joint work with Gabriel Currier and Chi Hoi Yip.

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# FAMILIES WITH LOWER BOUND ON THE SUM OF PAIRWISE INTERSECTIONS OF TRIPLES

8 Jul  
3:40pm  
Section 2

Kartal Nagy  
Eötvös University

We call a family  $\mathcal{F}$   $(3, 2, 1)$ -intersecting if  $|A \cap B| + |B \cap C| + |C \cap A| \geq l$  for all  $A, B, C \in \mathcal{F}$ . We try to look for the maximum size of such a family  $\mathcal{F}$  in case when  $\mathcal{F} \subset \binom{[n]}{k}$  or  $\mathcal{F} \subset 2^{[n]}$ .

In the uniform case we show that if  $\mathcal{F}$  is  $(3, 2, 2)$ -intersecting, then  $|\mathcal{F}| \leq \binom{n+1}{k-1} + \binom{n}{k-2}$  and if  $\mathcal{F}$  is  $(3, 2, 3)$ -intersecting, then  $|\mathcal{F}| \leq \binom{n}{k-1} + 2\binom{n}{k-3} + 3\binom{n-1}{k-3}$ . For the lower bound we construct a  $(3, 2, 1)$ -intersecting family and we show that this bound is sharp when  $l = 2$  or  $3$  and  $n$  is sufficiently large compared to  $k$ . In this case we show that  $|\mathcal{F}| = \binom{n-1}{k-1}$ . It can be seen that this theorem is a kind of sharpening of the Erdős-Ko-Rado theorem, since it gives the same upper bound for the corresponding pairs  $(n, k)$  under weaker conditions.

In the non-uniform case we give an upper bound for a  $(3, 2, n-x)$ -intersecting family, when  $n$  is sufficiently large compared to  $x$ . In this case the construction depends on the remainder of  $x$  by 6. If  $x = 6p + k$ , where  $0 \leq k \leq 5$ , then the optimal construction will include all sets with at least  $n-p$  elements and some sets with  $n-p-1$  and  $n-p-2$  elements, depending on the value of  $k$ .

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## Friendly partitions of regular graphs

11 Jul  
11:15am  
Section 2

Zoltán Lóránt Nagy  
Eötvös University

An *internal* or *friendly partition* of a graph is a partition of the vertices into two nonempty sets  $A \cup B$  so that every vertex has at least as many neighbours in its own class as in the other one. For regular graphs, this means  $|N(v) \cap A| \geq \deg(v)/2$  for all elements  $v \in A$  and  $|N(v) \cap B| \geq \deg(v)/2$  for all elements  $v \in B$ . An old conjecture attributed to DeVos states that for fixed  $d$ , such a partition exists for all  $d$ -regular graphs, apart from finitely many counterexamples. This has been resolved for  $d \in \{3, 4, 6\}$ . (We remark that this is also closely related to an conjecture of Zoltán Füredi, on friendly bisection of the Erdős-Rényi random graph).

In this talk I will give a survey of related results and highlight the following contributions, mostly related to the open  $d = 5$  case:

**Theorem 0.0.8** *In each  $n$ -vertex 5-regular graph, the intersection of 3-cohesive sets with minimum size contains at most  $n/4 + 1$  vertices.*

Here the term  $k$ -cohesive was introduced by Ban and Linial [?] for vertex sets spanning a graph of minimum degree at least  $k$ . Note that for 5-regular graph admitting friendly partitions, the intersection of 3-cohesive sets with minimum size contains exactly zero vertices. The result relies on the combination of algebraic techniques (the Alon-Friedland-Kalai theorem, an application of the Combinatorial Nullstellensatz) with combinatorial and random techniques.

**Theorem 0.0.9** *Asymptotically almost every 5-regular graph has an internal (friendly) partition.*

This would follow from a strengthening of the upper bound of the minimum bisection width problem for 5-regular random graphs, and refines the random process of Diaz, Serna and Wormald and their differential equation method as well.

Finally we study the problem for the incidence graphs of projective planes, show several connections and applications, and point out the possible room for improvement in finding partitions where the spanning graphs of the two parts have minimum degree greater than the half of the average degree.

The talk is based on a project joint work with P. Bärnkopf, Z. Paulovics, E. Csóka, P. Fekete, L. Szemerédi.

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## Higher rank antipodality

Márton Naszódi

HUN-REN Alfréd Rényi Institute of Mathematics

8 Jul  
3:40pm  
Section 1

Motivated by general probability theory, we say that the set  $X$  in  $\mathbb{R}^d$  is *antipodal of rank  $k$* , if for any  $k + 1$  elements  $q_1, \dots, q_{k+1} \in X$ , there is an affine map from  $\text{conv}X$  to the  $k$ -dimensional simplex  $\Delta_k$  that maps  $q_1, \dots, q_{k+1}$  onto the  $k + 1$  vertices of  $\Delta_k$ . For  $k = 1$ , it coincides with the well-studied notion of (pairwise) antipodality introduced by Klee. We consider the following natural generalization of Klee's problem on antipodal sets: What is the maximum size of an antipodal set of rank  $k$  in  $\mathbb{R}^d$ ? We present a geometric characterization of antipodal sets of rank  $k$  and adapting the argument of Danzer and Grünbaum originally developed for the  $k = 1$  case, we prove an upper bound which is exponential in the dimension. We point out that this problem can be connected to a classical question in computer science on finding perfect hashes, and it provides a lower bound on the maximum size, which is also exponential in the dimension.

Joint work with Zsombor Szilágyi and Mihály Weiner.

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## Monochromatic quadrilaterals in the max-norm plane

Alexander Natalchenko

Moscow Institute of Physics and Technology

11 Jul  
4:05pm  
Section 1

One of the open questions in Euclidean Ramsey theory on the plane is whether there is a sufficiently large  $m$  such that for every  $m$ -point set  $M$ , there exists a 2-coloring of the plane without monochromatic isometric copies of  $M$ . In the case when the Euclidean metric is replaced with the maximum metric, there are arbitrarily large sets that require at least 3 colors to avoid their monochromatic copies. So it is natural to ask if 3 colors are also sufficient for all sufficiently large sets. In this talk, we answer this question in the affirmative.

8 Jul  
4:40am  
Section 3

## On Asymptotic Local Turán Problems

Jiayi Nie

Fudan University

An  $r$ -uniform hypergraph has  $(q, p)$ -property if any set of  $q$  vertices spans a complete sub-hypergraph on  $p$  vertices. Let  $t_r(n, q, p)$  be the minimum edge density of an  $n$ -vertex  $r$ -uniform hypergraph with  $(q, p)$ -property and let  $t_r(q, p) = \lim_{n \rightarrow \infty} t_r(n, q, p)$ . A disjoint union of  $k$  complete hypergraphs has  $(q, \lceil q/k \rceil)$ -property, which gives  $t_r((q, \lceil q/k \rceil)) \leq 1/k^{r-1}$ . The Frankl, Huang and Rödl showed that these constructions are the best asymptotically, that is,  $\lim_{q \rightarrow \infty} t_r((q, \lceil q/k \rceil)) = 1/k^{r-1}$ . They asked whether it is true for all real number  $\gamma \geq 1$  that  $\lim_{q \rightarrow \infty} t_r((q, \lceil q/\gamma \rceil)) = 1/\lfloor \gamma \rfloor^{r-1}$ . In this paper, we give positive answers to this question for a small range of real numbers, and, on the other hand, provide new constructions that give negative answers for many other ranges. This is joint work with Péter Frankl.

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8 Jul  
4:40pm  
Section 1

## Forbidden complexes for the 3-sphere

Makoto Ozawa

Komazawa University

A simplicial complex is said to be *critical* (or *forbidden*) for the 3-sphere  $S^3$  if it cannot be embedded in  $S^3$  but after removing any one point, it can be embedded.

We show that if a multibranch surface cannot be embedded in  $S^3$ , it contains a critical complex which is a union of a multibranch surface and a (possibly empty) graph. We exhibit all critical complexes for  $S^3$  which are contained in  $K_5 \times S^1$  and  $K_{3,3} \times S^1$  families. We also classify all critical complexes for  $S^3$  which can be decomposed into  $G \times S^1$  and  $H$ , where  $G$  and  $H$  are graphs.

In spite of the above property, there exist complexes which cannot be embedded in  $S^3$ , but they do not contain any critical complexes. From the property of those examples, we define an equivalence relation on all simplicial complexes  $\mathcal{C}$  and a partially ordered set of complexes  $(\mathcal{C}/\sim; \subseteq)$ , and refine the definition of critical. According to the refined definition of critical, we show that if a complex  $X$  cannot be embedded in  $S^3$ , then there exists  $[X'] \subseteq [X]$  such that  $[X']$  is critical for  $[S^3]$ .

This is a joint work with Mario Eudave-Muñoz.

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## A generalized Ramsey-Turán problem

Cory Palmer

University of Montana

9 Jul  
12:05pm  
Section 2

The Ramsey-Turán problem for  $K_p$  asks for the maximum number of edges in an  $n$ -vertex  $K_p$ -free graph with independence number  $o(n)$ . A natural generalization counts cliques larger than the edge  $K_2$ . Let  $\mathbf{RT}(n, K_q, K_p, o(n))$  denote the maximum number of copies of  $K_q$  in an  $n$ -vertex  $K_p$ -free graph with independence number  $o(n)$ . Balogh, Liu and Sharifzadeh determined the asymptotics of  $\mathbf{RT}(n, K_3, K_p, o(n))$ . In this talk we will establish the asymptotics for counting copies of  $K_4$  and  $K_5$  and discuss the general problem for  $K_q$ .

Joint work with József Balogh and Van Magnan.

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## Coprime mappings and lonely runners

Fei Peng

National University of Singapore

11 Jul  
12:05pm  
Section 1

For  $x$  real, let  $\{x\}$  be the fractional part of  $x$  (i.e.  $\{x\} = x - \lfloor x \rfloor$ ). The lonely runner conjecture can be stated as follows: for any  $n$  positive integers  $v_1 < v_2 < \dots < v_n$  there exists a real number  $t$  such that  $1/(n+1) \leq \{v_i t\} \leq n/(n+1)$  for  $i = 1, \dots, n$ . We prove that if  $\epsilon > 0$  and  $n$  is sufficiently large (relative to  $\epsilon$ ) then such a  $t$  exists for any collection of positive integers  $v_1 < v_2 < \dots < v_n$  such that  $v_n < (2 - \epsilon)n$ . This is an approximate version of a natural next step for the study of the lonely runner conjecture suggested by Tao. The key ingredient in our proof is a result on coprime mappings. Let  $A$  and  $B$  be sets of integers. A bijection  $f : A \rightarrow B$  is a coprime mapping if  $a$  and  $f(a)$  are coprime for every  $a \in A$ . We show that if  $A, B \subset [n]$  are intervals of length  $2m$  where  $m = e^{\Omega((\log \log n)^2)}$  then there exists a coprime mapping from  $A$  to  $B$ . We do not believe that this result is sharp.

Joint work with Tom Bohman.

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## The Hamilton space of pseudorandom graphs

Kalina Petrova

ETH Zürich

12 Jul  
11:40am  
Section 1

We show that if  $n$  is odd and  $p \leq C \log n/n$ , then with high probability Hamilton cycles in  $G(n, p)$  span its cycle space. More generally, we show this holds for a class of graphs satisfying certain natural pseudorandom properties. The proof is based on a novel idea of parity-switchers, which can be thought of as analogues of absorbers in the context of cycle spaces. As another application of our method, we show that Hamilton cycles in a near-Dirac graph  $G$ , that is, a graph  $G$  with odd  $n$  vertices and minimum degree  $n/2 + C$  for sufficiently large constant  $C$ , span its cycle space. This is joint work with Micha Christoph and Rajko Nenadov.

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9 Jul  
11:40am  
Section 1

## **Erdős distinct distances problem, variants, and applications**

Thang Pham

Vietnam National University Hanoi

In this talk, I will present recent results on the Erdős distinct distances problem and its variants over arbitrary fields. Some applications on intersection patterns and incidence problems will be also discussed.

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11 Jul  
3:15pm  
Section 3

## **The maximum clique query problem**

András Pongrácz

HUN-REN Alfréd Rényi Institute of Mathematics

According to a classical result, the size of the maximum clique in  $G(n, 1/2)$  is asymptotically  $2 \log(n)$  with high probability, where  $\log$  is the base 2 logarithm. As an instance of the subgraph query problem, the following question arises: can we find such a large clique using a limited number of edge queries? This was recently answered in the negative: for any  $d < 2$ , there is a number  $a(d) < 2$  such that the size of the largest clique we can find using  $n^d$  edge queries is at most  $a(d) \log(n)$ . We show a more general estimate that provides an upper bound when queries are to be taken in a given number of rounds  $r$ . That is, there are  $r$  rounds, in each round we need to make queries simultaneously, only taking into consideration the results of previous rounds, and there is a limit  $n^d$  to the total number of queries. Furthermore, we generalize the results to the setup where the target subgraph is not necessarily a clique, but rather any subgraph of fixed minimum edge density. This is a joint work with Endre Csóka.

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9 Jul  
4:05pm  
Section 3

## **Positroid envelope classes and graphic positroids**

Jeremy Quail

University of Vermont

Postnikov developed a combinatorial decomposition of the Grassmannian using a class of ordered matroids, called positroids. Positroids are matroids realizable by real matrices with all nonnegative maximal minors. They partition the ordered matroids into equivalence classes, called positroid envelope classes. We prove that every positroid envelope class contains a graphic matroid. Then, we show that the following classes of positroids are equivalent: graphic, binary, and regular.

Joint with Puck Rombach.

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## A universal end space theory

Florian Reich  
Universitaet Hamburg

9 Jul  
3:40pm  
Section 2

We introduce a universal end space theory that unifies the existing end spaces of undirected and directed graphs and establishes end spaces for finitary matroids, hypergraphs and bidirected graphs.

Our main result shows that the tangle-like description of ends in undirected graphs, called directions, extends to this universal notion of ends: there is a one-to-one correspondence between “directions” and ends.

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## Odd Covers

Puck Rombach  
University of Vermont

11 Jul  
12:05pm  
Section 2

The “odd cover problem” of finding the minimum number of complete bipartite graphs, or bicliques, which cover every edge of the complete graph an odd number of times was proposed by Babai and Frankl in 1992. A more general question asks, given a simple graph  $G$ , for the minimum number of bicliques such that each edge of  $G$  is in an odd number of bicliques and each non-edge in an even number, denoted  $b_2(G)$ . We will talk about each of these problems. We provide new bounds for the odd cover problem, and solve the problem for complete graphs on  $n$  vertices when  $n$  is odd or a multiple of 8. This work was started at the Graduate Research Workshop in Combinatorics and is joint with Calum Buchanan, Alexander Clifton, Eric Culver, Jiaxi Nie, Jason O’Neill and Mei Yin.

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## Linear Turán numbers

Miklós Ruzinkó

HUN-REN Alfréd Rényi Institute of Mathematics & Sorbonne University Abu Dhabi

12 Jul  
10:00am  
Section 2

The Turán number of hypergraphs have been studied extensively. Here we explore a bit less studied subarea, the linear Turán number, and (mostly) we restrict ourselves to linear triple systems, a set of triples on a set of points in which any two triples intersect in at most one point. For a fixed linear triple system  $F$ , the linear Turán number  $\text{ex}_L(n, F)$  is the maximum number of triples in a linear triple system with  $n$  points that does not contain  $F$  as a subsystem. The case when  $C$  is the (linear) triangle, the famous result of Ruzsa and Szemerédi gives

$$n^{2-c\sqrt{\log(n)}} < \text{ex}_L(n, C) = o(n^2).$$

Here we explore a few cases when  $F$  is an acyclic triple system. We prove that for fixed  $k$  and large enough  $n$ ,  $\text{ex}_L(n, M_k) = f(n, k)$  where  $M_k$  is the set of  $k$  pairwise disjoint triples and  $f(k, n)$  is the maximum number of triples that can meet  $k - 1$

points in a linear triple system on  $n$  points. This is an analogue of an old result of Erdős on hypergraph matchings. For the  $k$ -edge linear path  $P_k$  we show that  $\text{ex}_L(n, P_k) \leq 1.75kn$ . We illustrate the difficulties by exploring  $\text{ex}_L(n, F)$  where  $F$  is a tree with at most four edges. These are joint results with A. Gyárfás and G.N. Sárközy, and Z. Füredi.

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11 Jul  
15:15pm  
Section 1

## Canonical theorems in Euclidean Ramsey theory

Arsenii Sagdeev

HUN-REN Alfréd Rényi Institute of Mathematics

We prove the following two results in Euclidean Ramsey theory. First, every coloring of the space, regardless of the number of colors used, contains either a monochromatic or a rainbow congruent copy of each acute triangle. Second, every coloring of  $R^n$  contains either a monochromatic or a rainbow congruent copy of an  $m$ -dimensional unit hypercube, provided that  $n$  is sufficiently large in terms of  $m$ . Joint work with Panna Gehér and Géza Tóth.

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9 Jul  
12:05pm  
Section 4

## THE DEADLOCK RESOLVING SETS OF KK-MBF CLASS, AND CARDINALITY ESTIMATES

Hasmik Sahakyan

Institute for Informatics and Automation Problems of NAS RA

In this paper, we study a special class of monotone Boolean functions coming from the theory of shadow minimization of finite set systems and known as the KK-MBF class of functions. In particular, we find upper and lower bounds for the unique deadlock resolving set for KK-MBF functions, and construct special class of KK-MBF functions for which the lower bound is achieved. Due to the resolving property, the estimate will show the number of required tests for query-based recognition of KK-MBF functions.

First, we introduce the notion of corner points for KK-MBF type functions.

**Definition 1.** A zero vertex  $\alpha$  of a  $f$  is called a *zero-corner point* if:

1.  $f = 1$  for all from the same layer, such that  $\beta <_{lex} \alpha$ , and
2.  $f = 1$  for all  $\beta, \alpha < \beta$  (component-wise order).

Similarly, a unit vertex of a KK-MBF type function  $f$  is called one-corner point if:

- $f = 0$  for all from the same layer such that  $\beta >_{lex} \alpha$ , and
- $f = 0$  for all  $\beta, \beta < \alpha$  (component-wise order).



Let  $z(f)$  denote the set of all zero-corner points, and  $o(f)$  denote the set of all one-corner points of function  $f$ .

**Proposition 1.** Every function  $f$  of class KK-MBF has a unique deadlock resolving set, which is:  $z(f) \cup o(f)$ .

**Proposition 2.** For any function  $f$  (defined in  $B^n, n \geq 3$ ) of class KK-MBF  $|z(f) \cup o(f)| \leq 2n - 2 + 1$ .

**Proposition 3.** There exists a function  $f$  (defined in  $B^n, n \geq 5$ ) in KK-MBF class, for which:  $|z(f) \cup o(f)| \geq 2(n - 3)$ .

## Stability results for forbidden configurations

Attila Sali

HUN-REN Alfréd Rényi Institute of Mathematics

12 Jul  
12:05pm  
Section 1

A matrix  $A$  is said to be *simple* if it is a  $(0,1)$ -matrix with no repeated columns. There is a natural correspondence between columns of  $A$  and subsets of  $[m]$ . We consider an extremal set problem in matrix terminology as follows. Let  $\|A\|$  be the number of columns of  $A$ . For a given matrix  $F$ , we say  $F$  is a *configuration* in  $A$  denoted  $F \prec A$  if there is a submatrix of  $A$  which is a row and column permutation of  $F$ . Define

$$\text{Avoid}(m, F) = \{A \mid A \text{ is } m\text{-rowed, } F \not\prec A\},$$

$$\text{forb}(m, F) = \max_{A \in \text{Avoid}(m, F)} \|A\|.$$

A matrix  $A \in \text{Avoid}(m, F)$  is called *extremal* if  $\|A\| = \text{forb}(m, F)$  and let

$$\text{ext}(m, F) = \{A \in \text{Avoid}(m, F) \mid \|A\| = \text{forb}(m, F)\}.$$

We seek to characterize extremal matrices by giving more information than just the definition. A number of the results had already been proven and in these cases the paper is more expository in nature. We also find some strong stability results, namely when a matrix  $A$  avoiding  $F$  is close to the bound, then some structures of the extremal matrices appear in  $A$ .

The results are joint work with Richard P. Anstee, Jaehwan Seok, Benjamin Kreiswirth and Bowen Li

## Linear three-uniform hypergraphs with no Berge path of given length

Nika Salia

King Fahd University of Petroleum and Minerals

12 Jul  
10:25am  
Section 2

Extensions of Erdős-Gallai Theorem for general hypergraphs are well studied. In this work, we prove the extension of Erdős-Gallai Theorem for linear hypergraphs. In particular, we show that the number of hyperedges in an  $n$ -vertex 3-uniform linear hypergraph, without a Berge path of length  $k$  as a subgraph is at most  $\frac{(k-1)}{6}n$  for  $k \geq 4$ . The bound is sharp for infinitely many  $k$  and  $n$ .

Joint work with: Ervin Győri

## Colored reachability in 3-quasi-transitive digraphs

Rocío Sánchez-López

UNAM

Let  $H$  be a digraph possibly with loops and  $D$  a digraph without loops whose arcs are colored with the vertices of  $H$  ( $D$  is said to be an  $H$ -colored digraph). A directed path  $P$  in  $D$  is said to be an  $H$ -path if and only if the consecutive colors encountered on  $P$  form a directed walk in  $H$ . An  $H$ -kernel of an  $H$ -colored digraph  $D$  is a subset of vertices of  $D$ , say  $N$ , such that for every pair of different vertices in  $N$  there is no  $H$ -path between them, and for every vertex  $u$  in  $V(D) - N$  there exists an  $H$ -path in  $D$  from  $u$  to  $N$ .  $D$  is said to be 3-quasi-transitive if for every pair of vertices  $u$  and  $v$  of  $D$ , the existence of a directed path of length 3 from  $u$  to  $v$  implies that  $\{(u, v), (v, u)\} \cap A(D) = \emptyset$ . In this talk we show a result regarding the existence of  $H$ -kernels in 3-quasi-transitive digraphs; mainly the existence of  $H$ -kernels is guaranteed by means of sufficient conditions on the directed cycles of length 3 and 4.

---

## On closed forms of C-recursive integer sequences

Lorenzo Sauras

IMAR

Mazzanti (in 2002) and Marchenkov (in 2007) achieved to prove, by improving Matiyasevich's methods, that every "usual" sequence of non-negative integers (or, more concretely, every Kalmár function) has a closed form.

Their notion of closed form is the so-called arithmetic term: an expression inductively constructed from positive integers and one variable, by applying sums, truncated subtractions, products, integer divisions and exponentiations.

In this talk, we will present a method to calculate arithmetic terms representing integer sequences that are given by a linear recurrence of constant integer coefficients, and we will provide several important examples.

This is a joint work with Mihai Prunescu.

---

# Repeatedly applying the Combinatorial Nullstellensatz for Zero-sum Grids to Martin Gardner's minimum no-3-in-a-line problem

John Schmitt

Middlebury College (USA)

12 Jul  
10:25am  
Section 1

A 1976 question of Martin Gardner asked for the minimum size of a placement of queens on an  $n \times n$  chessboard that is maximal with respect to the property of 'no-3-in-a-line'. When the lines are restricted to orthogonals and diagonals, the work of Cooper, Pikhurko, Schmitt and Warrington showed that this number is at least  $n$  in the cases that  $n \not\equiv 3 \pmod{4}$ , and at least  $n - 1$  in the case that  $n \equiv 3 \pmod{4}$ . When  $n > 1$  is odd, Gardner conjectured the lower bound to be  $n + 1$ . We prove this conjecture in the case that  $n \equiv 1 \pmod{4}$ . The proof relies heavily on a recent advancement to the Combinatorial Nullstellensatz for zero-sum grids due to Bogdan Nica.

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## Asymptotic normality for subgraph count in random (hyper)graphs

Gregorz Serafin

Wrocław University of Science and Tehnology

10 Jul  
11:40am  
Section 2

Let  $\mathbb{G}(n, p)$  be the binomial Erdős-Rényi random graph model and let  $\mathbb{H}(n, \mathbf{p})$ ,  $\mathbf{p}(n) = (p_1(n), \dots, p_n(n))$ , be its natural generalization to hypergraphs, where any hyperedge of size  $r$  exists, independently of other edges, with probability  $p_r(n)$ . For a fixed (hyper)graph  $G$  we denote by  $N_n^G$  the number of sub(hyper)graphs of  $\mathbb{G}(n, p)$  or  $\mathbb{H}(n, \mathbf{p})$  that are isomorphic to a fixed (hyper)graph  $G$  and by

$$\tilde{N}_n^G := \frac{N_n^G - \mathbb{E}[N_n^G]}{\sqrt{\text{Var}[N_n^G]}}$$

we denote its normalization. The problem we will examine is when and how fast does  $\tilde{N}_n^G$  converge to the standard normal distribution  $N(0, 1)$ . Concerning  $\mathbb{G}(n, p)$  and a graph  $G$ , necessary and sufficient conditions have been derived for  $\tilde{N}_n^G$  being asymptotically normal. This result has been then complemented with convergence rates in Wasserstein distance. During the talk we will present a new, very short proof of the necessary conditions and establish convergence rates in Kolmogorov distance, which is simply the  $L^\infty$  distance between distribution functions:

$$d_K(\tilde{N}_n^G, N(0, 1)) := \sup_{x \in \mathbb{R}} |P(\tilde{N}_n^G \leq x) - P(N(0, 1) \leq x)|.$$

The main focus of the talk, however, will be devoted to discussing analogous problems in the random hypergraph model  $\mathbb{H}(n, \mathbf{p})$ . An especially interesting case is when some edge probabilities tend to 0, and some to 1. Additionally, we will show a connection of the presented results to the so called *fourth moment phenomenon*.

---

## Degree conditions restricted to induced Net and Wounded for hamiltonicity of claw- $o$ -heavy graphs

8 Jul  
3:40pm  
Section 3

Wangyi Shang

Northwestern Polytechnical University

We say that a graph  $G$  is  $\{H, F\}$ - $o$ -heavy if every induced subgraph of  $G$  isomorphic to  $H$  or  $F$  contains two nonadjacent vertices with degree sum at least  $|V(G)|$ . In 2012, Li et al. proved that 2-connected  $\{K_{1,3}, N\}$ - $o$ -heavy and 2-connected  $\{K_{1,3}, W\}$ - $o$ -heavy graph are hamiltonian. In this paper, we further give some Ore-type conditions restricting to induced copies of  $N$  and  $W$  of a 2-connected claw- $o$ -heavy graph that can guarantee the graph to be hamiltonian. This improves some previous related results.

---

## Star colouring of circulant graphs

9 Jul  
12:05pm  
Section 3

Yueping Shi

Sun Yat-sen University

A *star colouring* of a graph  $G$  is a proper vertex-colouring such that any two colour classes induce a star forest. Equivalently, no path of length 3 in  $G$  is 2-coloured. The *star chromatic number*  $\chi_{st}(G)$  of  $G$  is the minimum number of colours in a star colouring of  $G$ . This model of vertex-colouring was suggested by Grünbaum in 1973, and has attracted significant attention since around 2004.

For  $D \subset \{1, 2, \dots, \lfloor \frac{n}{2} \rfloor\}$ , the *circulant graph* of order  $n$ , generated by  $D$ , is the graph  $C(n; D)$  with vertex set  $\{v_0, \dots, v_{n-1}\}$  (indices taken modulo  $n$ ), where for all  $0 \leq i \leq n-1$ , we have  $v_i v_{i+j}$  and  $v_i v_{i-j}$  are edges if and only if  $j \in D$ . Some special cases of circulant graphs include:

- Generalised Andrásfai graphs  $A_k(s, t)$ , where  $n = t(k-1) + 2s$  with  $k \geq 1$  and  $0 \leq s < t$ , and  $D$  consists of the integers  $\equiv s \pmod{t}$ . The graphs  $A_k(1, 3)$  are the Andrásfai graphs.
- Harary graphs, where  $2k < n$ .  $H_{2k,n} = C(n; \{1, \dots, k\})$ . If  $n$  is even,  $H_{2k+1,n} = C(n; \{1, \dots, k, \frac{n}{2}\})$ .

The study of the classical chromatic number of circulant graphs has attracted significant interest. In this talk, we shall present some results for the star chromatic number of some classes of circulant graphs, including generalised Andrásfai graphs and Harary graphs. Joint work with Henry Liu (Sun Yat-sen University).

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## Sidorenko-type inequalities for Trees

Lina Simbaqueba  
University of Vitoria

8 Jul  
4:15pm  
Section 4

Given two graphs  $H$  and  $G$ , the homomorphism density  $t(H, G)$  represents the likelihood that a random mapping from  $V(H)$  to  $V(G)$  is a homomorphism. Sidorenko Conjecture states that for any bipartite graph  $H$ ,  $t(H, G)$  is greater or equal to  $t(K_2, G)^{e(H)}$ .

Introducing a binary relation  $H \geq T$  if and only if  $t(H, G)^{e(T)} \geq t(T, G)^{e(H)}$  for all graphs  $G$ , we establish a partial order on the set of non-empty connected graphs. Employing a technique by Kopparty and Rossman, which involves the use of entropy to define a linear program, we derive several necessary and sufficient conditions for two trees  $T, F$  satisfy  $T \geq F$ . Furthermore, we show how important results and open problems in extremal graph theory can be reframed using this binary relation.

Joint work with Natalie Behage, Gabriel Crudele, and Jonathan Noel.

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## Chromatic number: Problems, puzzles, and paradoxes

Vaidy Sivaraman  
Mississippi State University

9 Jul  
11:15am  
Section 3

The chromatic number of a graph is an invariant of fundamental importance in structural and algorithmic graph theory. How does a graph look like if it has large chromatic number? What structures can we find in it? This talk will feature several open problems relating the chromatic number to induced subgraphs, minors, and graph complementation.

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## Forbidden patterns among grid-points, hypergraphs and geometric arrangements

József Solymosi  
University of British Columbia

9 Jul  
10:00am  
Section 1

What is the maximum density of a subset of the  $[n] \times [n]$  integer grid avoiding a given pattern? In some cases, it is just  $o(n^2)$ , while for others, it is much smaller,  $O(n^{2-c})$ . Characterizing the difference between them is a difficult task. We will see examples of situations where analyzing hypergraphs based on the configuration of the points can help us get the right answer. Similar problems will be discussed when a point-line arrangement forbidding an incidence sub-structure forces the maximum number of incidences to be much smaller than in extremal constructions. We show the connection between the three types of questions, proving new results and listing several open problems.

---

10 Jul  
11:15am  
Section 2

## The random Turán problem

Sam Spiro  
Rutgers University

Let  $G_{n,p}$  denote the random  $n$ -vertex graph obtained by including each edge independently and with probability  $p$ . Given a graph  $F$ , let  $\text{ex}(G_{n,p}, F)$  denote the size of a largest  $F$ -free subgraph of  $G_{n,p}$ . When  $F$  is non-bipartite, the asymptotic behavior of  $\text{ex}(G_{n,p}, F)$  was determined in breakthrough work done independently by Conlon-Gowers and by Schacht. In this talk we discuss some recent results for bipartite  $F$  (where much less is known), as well as for the analogous problem for  $r$ -partite  $r$ -graphs.

---

10 Jul  
11:15am  
Section 2

## Criticality in Sperner's lemma

Matej Stehlik  
Université Paris Cité / IRIF

Sperner's lemma states that in any labelling of the vertices of a triangulation of a  $d$ -simplex with  $d + 1$  labels, such that each vertex of the  $d$ -simplex receives a distinct label and any vertex lying in a face of the  $d$ -simplex has the same label as one of the vertices of that face, there exists a "rainbow facet" (one whose vertices have pairwise distinct labels). Tibor Gallai proved, in a different but equivalent form, that for  $d = 1$  and  $d = 2$ , we can pick any facet of the triangulation and find a labelling where that facet is the unique rainbow facet. In this talk, I will show that this does not hold in higher dimensions, thereby giving a negative answer to a question of Gallai from 1969. The construction is based on the properties of a neighbourly 4-polytope described by Grünbaum.

Joint work with Tomas Kaiser and Riste Skrekovski.

---

12 Jul  
11:40am  
Section 3

## The structure of directed 1-separations in directed graphs with cyclic torsoids

Qiuzhenyu Tao  
University of Hamburg

Graph separations are a fundamental property that has fascinated mathematicians for a long time. For small values of  $k$ , there exist simple and canonical combinatorial structures that exhibit all  $k$ -separations of a  $k$ -connected undirected graph, along with their inter-relationships. For  $k = 1$ , this is called the block-cut decomposition, and for  $k = 2$ , it's known as the Tutte decomposition. However, for directed graphs, very little is known. Even for  $k = 1$ , previous progress was limited to a partial result by Lovász. We will discuss recent progress related to directed 1-separations of directed graphs with cyclic torsoids.

---

## On vertices of the polytope of polystochastic matrices

Anna Taranenko

Sobolev Institute of Mathematics, Novosibirsk, Russia

10 Jul  
12:05pm  
Section 4

We will say that a multidimensional matrix  $A$  is polystochastic if it has nonnegative entries and the sum of entries in each line is equal to 1. Two-dimensional polystochastic matrices are usually called doubly stochastic. A set of  $d$ -dimensional polystochastic matrices of order  $n$  is a polytope  $\Omega_n^d$ .

The well-known Birkhoff theorem states that all vertices of the polytope of doubly stochastic matrices are permutation matrices. While for  $d \geq 3$  multidimensional permutations, i.e., polystochastic  $(0, 1)$ -matrices, are also vertices of the polytope  $\Omega_n^d$ , there exist many other vertices different from permutations.

In this talk, we review known bounds on the numbers of vertices of  $\Omega_n^d$  and propose iterative constructions of vertices  $\Omega_n^d$  based on products of multidimensional matrices. Next, we pay special attention to the polytope of polystochastic matrices of order 3. In particular, we find all vertices of the polytope of 4-dimensional polystochastic matrices of order 3 and show that for every  $d \geq 4$  there is a vertex of  $\Omega_3^d$  with a large support.

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## Sharp stability for the Brunn-Minkowski inequality for arbitrary sets

Marius Tiba

University of Oxford

11 Jul  
11:40am  
Section 1

The Brunn-Minkowski inequality states that for (open) sets  $A$  and  $B$  in  $R^d$ , we have  $|A + B|^{1/d} \geq |A|^{1/d} + |B|^{1/d}$ . Equality holds if and only if  $A$  and  $B$  are convex and homothetic sets in  $R^d$ . In this talk, we present a sharp stability result for the Brunn-Minkowski inequality, concluding a long line of research on this problem. We show that if we are close to equality in the Brunn-Minkowski inequality, then  $A$  and  $B$  are close to being homothetic and convex, establishing the exact dependency between the three notions of closeness.

Joint work with Alessio Figalli, Peter van Hintum.

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## Dedekind's problem in the hypergrid

István Tomon

Umea University

9 Jul  
3:40pm  
Section 1

One of the oldest problems in enumerative combinatorics is Dedekind's problem from 1897, asking for the number of antichains in the Boolean lattice. A natural generalization of Dedekind's problem asks for the number of antichains in the hypergrid  $\{1, \dots, t\}^n$ . The problem of estimating this number gained a lot of interest in the past decade due to its connection to certain hypergraph Ramsey problems. I will talk about our recent result with Victor Falgas-Ravry and Eero Rätty, which establishes a sharp estimate for every  $t$  and  $n$ , improving on all previous works.

8 Jul  
11:25am  
Section 2

## Disjoint list-colorings for planar graphs

Wouter Cames van Batenburg  
Umea University

It is well-known that every planar graph is 4-colorable, 5-choosable, and in fact admits exponentially many L-colorings for every 5-list-assignment L. We go beyond mere counting and are interested in a 'balanced' collection of L-colorings. We show that every 8-list-assignment L of a planar graph admits 8 L-colorings such that at every vertex  $v$ , every color in its list  $L(v)$  occurs in precisely one of these L-colorings. In other words, there exists 8 disjoint L-colorings.

We explore the same question under various girth constraints, for small excluded minors, and for bounded maximum average degree. For instance, the above holds more generally for every graph with maximum average degree smaller than 6. Our results remain true for correspondence coloring instead of list coloring, and they also have immediate implications for a flexible notion of list-coloring that was introduced by Dvořák, Norin and Postle (List coloring with requests, JGT, 2019). More precisely: suppose every  $k$ -list assignment admits  $k$  disjoint  $L$ -colorings, and furthermore every vertex has a preferred color from its list. Then there exists an  $L$ -coloring that agrees with at least a fraction  $1/k$  of those preferences.

Based on joint work with Stijn Cambie and Xuding Zhu

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11 Jul  
11:15am  
Section 1

## Ruzsa's discrete Brunn-Minkowski inequality and locality in sumsets.

Peter van Hintum  
University of Oxford

We explore higher dimensional additive phenomena in the integers proving optimal bounds in Freiman type results. As an application of the technical framework we prove a Brunn-Minkowski type inequality conjectured by Ruzsa asserting the following. For all  $d, t$ , and  $a \geq 0$ , there exists an  $n = O_{d,t}(1/a)$  so that given sets of integers  $A$  and  $B$  so that  $B$  is not contained efficiently in  $n$  generalised arithmetic progressions of dimension  $(d-1)$  and  $t|A| < |B| < |A|/t$ , we have  $|A+B|^{1/d} > |A|^{1/d} + (1-a)|B|^{1/d}$ .

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## The fractional chromatic number of the plane is at least 4

Dániel Varga

HUN-REN Alfréd Rényi Institute of Mathematics

10 Jul  
11:40am  
Section 1

We prove that the fractional chromatic number (FCN) of the unit distance graph of the Euclidean plane is greater than or equal to 4. A fundamental ingredient of the proof is the notion of geometric fractional chromatic number (GFCN) introduced recently by Ambrus et al. First, we establish that the GFCN of the plane is equal to its FCN by exploiting the amenability of the group of Euclidean transformations in dimension 2. Second, we provide a specific unit distance graph  $G$  on 27 vertices such that its GFCN is exactly 4.

Preprint: <https://arxiv.org/abs/2311.10069>

Joint work with Máté Matolcsi, Imre Z. Ruzsa, and Pál Zsámboki.

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## Trails in arc-colored digraphs with restriction in the color transitions

Carlos Vilchis-Alfaro

Instituto de Matematicas, UNAM

12 Jul  
11:40am  
Section 2

Edge-colored (di)graphs have proved to be very useful along many years for modeling problems in many areas, e.g., molecular biology, physical sciences, social sciences, among others. In particular, walks with a specific color pattern (such as, properly colored, monochromatic or rainbow) have been essential to solve these problems.

Let  $D$  be a digraph without loops, and  $H$  a digraph possibly with loops. An  $H$ -coloring of  $D$  is a function  $c : A(D) \rightarrow V(H)$ . We say that  $D$  is an  $H$ -colored digraph whenever we are taking a fixed  $H$ -coloring of  $D$ . A trail  $W = (v_0, e_0, v_1, e_1, v_2, \dots, v_{n-1}, e_{n-1}, v_n)$  in  $D$  is an  $H$ -trail if and only if  $(c(e_i), c(e_{i+1}))$  is an arc in  $H$ , for each  $i \in \{0, \dots, n-2\}$ . In this talk we will discuss the existence of eulerian  $H$ -trails and deal with the complexity of finding  $s-t$   $H$ -trails and  $H$ -paths in  $H$ -colored digraphs.

---

## The number of spanning trees in 4-regular simple graphs

Zezealem Yilma

Carnegie Mellon University Qatar

10 Jul  
11:40am  
Section 3

Extending an earlier work by Kostochka for subcubic graphs, we show that a connected graph  $G$  with minimum degree 2 and maximum degree 4 has at least  $75^{\frac{n_4}{5} + \frac{n_3}{10} + \frac{1}{5}}$  spanning trees, where  $n_i$  is the number of vertices of degree  $i$  in  $G$ , unless  $G$  is the complete graph on 5 vertices or obtained from the complete graph on 6 vertices by deleting the edges of a perfect matching. This, in particular, allows us to determine the value of the inferior limit of the normalised number of spanning trees (introduced by Alon) over the class of connected 4-regular graphs to be  $75^{1/5}$ .

Joint work with Jean-Sébastien Sereni.

# PALEY-LIKE QUASI-RANDOM GRAPHS ARISING FROM POLYNOMIALS

8 Jul  
3:40pm  
Section 4

Semin Yoo  
IBS

Paley graphs and Paley sum graphs are classical examples of quasi-random graphs. In this talk, we provide new constructions of families of quasi-random graphs that behave like Paley graphs but are neither Cayley graphs nor Cayley sum graphs. These graphs give a unified perspective of studying various graphs defined by polynomials over finite fields such as Paley graphs, Paley sum graphs, and graphs coming from Diophantine tuples and their generalizations. We also provide new lower bounds on the clique number and independence number of general quasi-random graphs. In particular, we give a sufficient condition for the clique number of quasi-random graphs of order  $n$  to be at least  $(\log_{3.008} - o(1))n$ . Such a condition applies to many classical quasi-random graphs as well as some new graphs we construct, including Paley graphs and Paley sum graphs. This is joint work with Seouyoung Kim, and Chi Hoi Yip.

---

## On $\mathcal{F}$ -convexity

8 Jul  
11:25am  
Section 4

Liping Yuan  
Hebei Normal University

Let  $\mathcal{F}$  be a family of sets in  $\mathbb{R}^d$ . A set  $M \subset \mathcal{F}$  is called  $\mathcal{F}$ -convex if for any pair of distinct points  $x, y \in M$ , there is a set  $F \in \mathcal{F}$  such that  $x, y \in F$  and  $F \subset M$ . In this talk we'll discuss  $\mathcal{F}$ -convexity and related problems for some interesting families  $\mathcal{F}$ , including characterizations of  $\mathcal{F}$ -convex sets,  $\mathcal{F}$ -convex completions, generic  $\mathcal{F}$ -convexity,  $\mathcal{F}$ -convex functions and so on.

---

## The absence of monochromatic triangles implies various properly colored spanning trees

11 Jul  
3:40pm  
Section 4

Shenggui Zhang  
Northwestern Polytechnical University, Xi'an

An edge-colored graph  $G$  is called properly colored if every two adjacent edges are assigned distinct colors. A monochromatic triangle is a cycle of length 3 with the edges assigned a same color. Given a tree  $T_0$ , let  $\mathcal{T}(n, T_0)$  be the collection of  $n$ -vertex trees that are subdivisions of  $T_0$ . It is conjectured that for each fixed tree  $T_0$  of  $k$  edges, there is a function  $f(k)$  such that for each integer  $n > f(k)$  and each  $T \in \mathcal{T}(n, T_0)$ , every edge-colored complete graph  $K_n$  without containing any monochromatic triangle must contain a properly colored copy of  $T$ . The case that  $T_0$  is a star is confirmed. A weaker version of the above conjecture is also obtained. Moreover, to get a nice quantitative estimation of  $f(k)$  requires determining the Constraint Ramsey number of a monochromatic triangle and a rainbow  $k$ -star, which is of independent interest.

---

# ON A VARIANT OF THE NARKIEWICZ CONSTANT OF FINITE ABELIAN GROUPS

QINGHAI ZHONG

University of Graz

8 Jul  
12:15pm  
Section 3

A typical zero-sum problem studies conditions which ensure that given sequences have nontrivial zero-sum subsequences with prescribed property. For example, the Davenport constant and the Erdős- Ginzberg-Ziv constant are classical zero-sum constants. A natural generalization is to study conditions which ensure that given sequences  $S$  have two nontrivial zero-sum subsequences with prescribed properties and relations. We introduced new invariants  $D^N(G)$  and  $\eta^N(G)$ , and proved that  $D^N(G) + 1$  is just another description of the Narkiewicz constant  $N_1(G)$ , which was first used by Narkiewicz in 1960 to study the asymptotic behavior of counting functions associated with non-unique factorizations. We will talk about the direct and inverse problems of  $D^N(G)$  and  $\eta^N(G)$ .

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